INTERNATIONAL ADVANCED LEVEL

MATHEMATICS/ FURTHER MATHEMATICS/ PURE MATHEMATICS

SPECIFICATION

Pearson Edexcel International Advanced Subsidiary in Mathematics (XMA01)
Pearson Edexcel International Advanced Subsidiary in Further Mathematics (XFM01)
Pearson Edexcel International Advanced Subsidiary in Pure Mathematics (XPM01)
Pearson Edexcel International Advanced Level in Mathematics (YMA01)
Pearson Edexcel International Advanced Level in Further Mathematics (YFM01)
Pearson Edexcel International Advanced Level in Pure Mathematics (YPM01)

First teaching September 2018
First examination from January 2019
First certification from August 2019 (International Advanced Subsidiary) and August 2020 (International Advanced Level)
Issue 3

Edexcel, BTEC and LCCI qualifications

Edexcel, BTEC and LCCI qualifications are awarded by Pearson, the UK's largest awarding body offering academic and vocational qualifications that are globally recognised and benchmarked. For further information, please visit our qualification website at qualifications.pearson.com. Alternatively, you can get in touch with us using the details on our contact us page at qualifications.pearson.com/contactus

About Pearson

Pearson is the world's leading learning company, with 35,000 employees in more than 70 countries working to help people of all ages to make measurable progress in their lives through learning. We put the learner at the centre of everything we do, because wherever learning flourishes, so do people. Find out more about how we can help you and your learners at qualifications.pearson.com

Acknowledgements

This specification has been produced by Pearson on the basis of consultation with teachers, examiners, consultants and other interested parties. Pearson would like to thank all those who contributed their time and expertise to the specification's development.

ISBN 978 1 446 94981 8

Summary of Pearson Edexcel International Advanced Subsidiary/Advanced Level in Mathematics, Further Mathematics and Pure Mathematics Specification Issue 3 changes

<table>
<tr><th>Summary of changes made between previous issue and this current issue</th><th>Page number</th></tr>
<tr><td>In section Notation and formulae, Integration, the equation for a^x now reads as follows:

$a^x \qquad \frac{a^x}{\ln a} + c$</td><td>22</td></tr>
<tr><td>In Unit P4.3, the following section 6.5 has been added for clarification:
<table><tr><td>6.5</td><td>Use integration to find the area under a curve given its parametric equations.</td><td>Students should be able to find the area under a curve given its parametric equations. Students will not be expected to sketch a curve from its parametric equations.</td></tr></table></td><td>28</td></tr>
</table>

Earlier issues show previous changes.

If you need further information on these changes or what they mean, contact us via our website at: qualifications.pearson.com/en/support/contact-us.html.

Contents

About this specification

The Pearson Edexcel International Advanced Subsidiary in Mathematics, Further Mathematics and Pure Mathematics and the Pearson Edexcel International Advanced Level in Mathematics, Further Mathematics and Pure Mathematics and are part of a suite of International Advanced Level qualifications offered by Pearson.

These qualifications are not accredited or regulated by any UK regulatory body.

Key features

This specification includes the following key features.

Structure

The Pearson Edexcel International Advanced Subsidiary in Mathematics, Further Mathematics and Pure Mathematics and the Pearson Edexcel International Advanced Level in Mathematics, Further Mathematics and Pure Mathematics are modular qualifications.

The Advanced Subsidiary and Advanced Level qualifications can be claimed on completion of the required units, as detailed in the *Qualification overview* section.

Content

- A variety of 14 equally weighted units allowing many different combinations, resulting in flexible delivery options.
- Core mathematics content separated into four Pure Mathematics units.
- From the legacy qualification:
    - Decision Mathematics 1 has been updated for a more balanced approach to content.
    - The Further, Mechanics and Statistics units have not changed.

Assessment

- Fourteen units tested by written examination.
- Pathways leading to International Advanced Subsidiary Level and International Advanced Level in Mathematics, Further Mathematics and Pure Mathematics.

Approach

Students will be encouraged to take responsibility for their own learning and mathematical development. They will use their knowledge and skills to apply mathematics to real-life situations, solve unstructured problems and use mathematics as an effective means of communication.

Specification updates

This specification is Issue 3 and is valid for first teaching from September 2018. If there are any significant changes to the specification, we will inform centres in writing. Changes will also be posted on our website.

For more information please visit qualifications.pearson.com

Using this specification

This specification gives teachers guidance and encourages effective delivery. The following information will help teachers to get the most out of the content and guidance.

Compulsory content: as a minimum, all the bullet points in the content must be taught. The word 'including' in content specifies the detail of what must be covered.

Examples: throughout the content, we have included examples of what could be covered or what might support teaching and learning. It is important to note that examples are for illustrative purposes only and that centres can use other examples. We have included examples that are easily understood and recognised by international centres.

Assessments: use a range of material and are not limited to the examples given. Teachers should deliver these qualifications using a good range of examples to support the assessment of the content.

Depth and breadth of content: teachers should use the full range of content and all the assessment objectives given in the subject content section.

Qualification aims and objectives

The aims and objectives of these qualifications are to enable students to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment
- develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs
- extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems
- develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected
- recognise how a situation may be represented mathematically and understand the relationship between 'real-world' problems and standard and other mathematical models and how these can be refined and improved
- use mathematics as an effective means of communication
- read and comprehend mathematical arguments and articles concerning applications of mathematics
- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations
- develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

Qualification abbreviations used in this specification

The following abbreviations appear in this specification:

International Advanced Subsidiary – IAS

International A2 – IA2

International Advanced Level – IAL

Why choose Edexcel qualifications?

Pearson – the world's largest education company

Edexcel academic qualifications are from Pearson, the UK's largest awarding organisation. With over 3.4 million students studying our academic and vocational qualifications worldwide, we offer internationally recognised qualifications to schools, colleges and employers globally.

Pearson is recognised as the world's largest education company, allowing us to drive innovation and provide comprehensive support for Edexcel students to acquire the knowledge and skills they need for progression in study, work and life.

A heritage you can trust

The background to Pearson becoming the UK's largest awarding organisation began in 1836, when a royal charter gave the University of London its first powers to conduct exams and confer degrees on its students. With over 150 years of international education experience, Edexcel qualifications have a firm academic foundation, built on the traditions and rigour associated with Britain's educational system.

To find out more about our Edexcel heritage please visit our website: qualifications.pearson.com/en/about-us/about-pearson/our-history

Results you can trust

Pearson's leading online marking technology has been shown to produce exceptionally reliable results, demonstrating that at every stage, Edexcel qualifications maintain the highest standards.

Developed to Pearson's world-class qualifications standards

Pearson's world-class standards mean that all Edexcel qualifications are developed to be rigorous, demanding, inclusive and empowering. We work collaboratively with a panel of educational thought-leaders and assessment experts to ensure that Edexcel qualifications are globally relevant, represent world-class best practice and maintain a consistent standard.

For more information on the world-class qualification process and principles please go to *Appendix 2: Pearson World Class Qualification design principles* or visit our website: uk.pearson.com/world-class-qualifications.

Why choose Pearson Edexcel International Advanced Subsidiary/Advanced Level qualifications in Mathematics, Further Mathematics and Pure Mathematics?

We have listened to feedback from all parts of the international school subject community, including a large number of teachers. We have made changes that will engage international learners and give them skills that will support their progression to further study of mathematics and to a wide range of other subjects.

Key qualification features – Unitised structure with all units equally weighted, allowing many different combinations of units and greater flexibility. Three exam series per year means students can sit unit exams when they are ready.

Clear and straightforward question papers – our question papers are clear and accessible for students of all ability ranges. Our mark schemes are straightforward so that the assessment requirements are clear.

Broad and deep development of learners' skills – we designed the International Advanced Level qualifications to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment
- develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs
- extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems
- develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected
- recognise how a situation may be represented mathematically and understand the relationship between 'real-world' problems and standard and other mathematical models and how these can be refined and improved
- use mathematics as an effective means of communication
- read and comprehend mathematical arguments and articles concerning applications of mathematics
- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations
- develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

Progression – International Advanced Level qualifications enable successful progression to H.E. courses in mathematics and many other subjects and to employment. Through our world-class qualification development process we have consulted with higher education to validate the appropriateness of these qualifications, including content, skills and assessment structure.

More information can be found on our website (qualifications.pearson.com) on the Edexcel International Advanced Level pages.

Supporting you in planning and implementing these qualifications

Planning

- Our *Getting Started Guide* gives you an overview of the Pearson Edexcel International Advanced Subsidiary/Advanced Level in Mathematics qualifications to help you understand the changes to content and assessment, and what these changes mean for you and your students.
- We will provide you with an editable course planner and scheme of work.
- Our mapping documents highlight key differences between the new and legacy qualifications.

Teaching and learning

- Print and digital learning and teaching resources – promote any time, any place learning to improve student motivation and encourage new ways of working.

Preparing for exams

We will also provide a range of resources to help you prepare your students for the assessments, including:

- specimen papers to support formative assessments and mock exams
- examiner commentaries following each examination series.

ResultsPlus

ResultsPlus provides the most detailed analysis available of your students' examination performance. It can help you identify the topics and skills where further learning would benefit your students.

examWizard

A free online resource designed to support students and teachers with examination preparation and assessment.

Training events

In addition to online training, we host a series of training events each year for teachers to deepen their understanding of our qualifications.

Get help and support

Our subject advisor service will ensure that you receive help and guidance from us. You can sign up to receive email updates from Graham Cumming's famous maths emporium for qualification updates and product and service news.

Just email mathsemporium@pearson.com and ask to be included in the email updates.

Qualification at a glance

Qualification overview

This specification contains the units for the following qualifications:

- Pearson Edexcel International Advanced Subsidiary/Advanced Level in Mathematics
- Pearson Edexcel International Advanced Subsidiary/Advanced Level in Further Mathematics
- Pearson Edexcel International Advanced Subsidiary/Advanced Level in Pure Mathematics

Course of study

The structure of these qualifications allows teachers to construct a course of study that can be taught and assessed as either:

- distinct units of teaching and learning with related assessments taken at appropriate stages during the course; or
- a linear course assessed in its entirety at the end.

Students study a variety of units, following pathways to their desired qualification.

Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.

Content and assessment overview

Each unit:

- is externally assessed
- has a written examination of 1 hour and 30 minutes
- has 75 marks.

Unit	*Unit code:	Availability	First assessment	IAS weighting	IAL weighting	Content overview
Pure Mathematics units						
P1: Pure Mathematics 1	WMA11/01	January, June and October	January 2019	33⅓ %	16⅔ %	Algebra and functions; coordinate geometry in the (x, y); trigonometry; differentiation; integration.
P2: Pure Mathematics 2	WMA12/01	January, June and October	June 2019	33⅓ %	16⅔ %	Proof; algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; exponentials and logarithms; trigonometry; differentiation; integration.
P3: Pure Mathematics 3	WMA13/01	January, June and October	January 2020	N/A	16⅔ %	Algebra and functions; trigonometry; exponentials and logarithms; differentiation; integration; numerical methods.
P4: Pure Mathematics 4	WMA14/01	January, June and October	June 2020	N/A	16⅔ %	Proof; algebra and functions; coordinate geometry in the (x, y) plane; binomial expansion; differentiation; integration; vectors.
FP1: Further Pure Mathematics 1	WFM01/01	January and June	June 2019	33⅓ %	16⅔ %	Complex numbers; roots of quadratic equations; numerical solution of equations; coordinate systems; matrix algebra; transformations using matrices; series; proof.

Unit	*Unit code:	Availability	First assessment	IAS weighting	IAL weighting	Content overview
FP2: Further Pure Mathematics 2	WFM02/01	January and June	June 2020	33⅓ %	16⅔ %	Inequalities; series; further complex numbers; first order differential equations; second order differential equations; Maclaurin and Taylor series; Polar coordinates.
FP3: Further Pure Mathematics 3	WFM03/01	January and June	June 2020	33⅓ %	16⅔ %	Hyperbolic functions; further coordinate systems; differentiation; integration; vectors; further matrix algebra.
Applications units						
M1: Mechanics 1	WME01/01	January, June and October	June 2019	33⅓ %	16⅔ %	Mathematical models in mechanics; vectors in mechanics; kinematics of a particle moving in a straight line; dynamics of a particle moving in a straight line or plane; statics of a particle; moments.
M2: Mechanics 2	WME02/01	January, June and October	June 2020	33⅓ %	16⅔ %	Kinematics of a particle moving in a straight line or plane; centres of mass; work and energy; collisions; statics of rigid bodies.
M3: Mechanics 3	WME03/01	January and June	June 2020	33⅓ %	16⅔ %	Further kinematics; elastic strings and springs; further dynamics; motion in a circle; statics of rigid bodies.
S1: Statistics 1	WST01/01	January, June and October	June 2019	33⅓ %	16⅔ %	Mathematical models in probability and statistics; representation and summary of data; probability; correlation and regression; discrete random variables; discrete distributions; the Normal distribution.
S2: Statistics 2	WST02/01	January, June and October	June 2020	33⅓ %	16⅔ %	The Binomial and Poisson distributions; continuous random variables; continuous distributions; samples; hypothesis tests.

Unit	*Unit code:	Availability	First assessment	IAS weighting	IAL weighting	Content overview
S3: Statistics 3	WST03/01	January and June	June 2020	33⅓ %	16⅔ %	Combinations of random variables; sampling; estimation, confidence intervals and tests; goodness of fit and contingency tables; regression and correlation.
D1: Decision Mathematics 1	WDM11/01	January and June	June 2019	33⅓ %	16⅔ %	Algorithms; algorithms on graphs; algorithms on graphs II; critical path analysis; linear programming.

*See *Appendix 1: Codes* for a description of this code and all other codes relevant to these qualifications.

Qualification overview

Pearson Edexcel International Advanced Subsidiary

The International Advanced Subsidiary in Mathematics, Further Mathematics and Pure Mathematics qualifications each consist of three externally-examined units:

Qualification	Compulsory units	Optional units
International Advanced Subsidiary in Mathematics	P1, P2	M1, S1, D1
International Advanced Subsidiary in Further Mathematics	FP1	FP2, FP3, M1, M2, M3, S1, S2, S3, D1
International Advanced Subsidiary in Pure Mathematics	P1, P2, FP1	

Pearson Edexcel International Advanced Level

The International Advanced Level in Mathematics, Further Mathematics and Pure Mathematics qualifications each consist of six externally-examined units:

Qualification	Compulsory units	Optional units
International Advanced Level in Mathematics	P1, P2, P3, P4	M1 and S1 or M1 and D1 or M1 and M2 or S1 and D1 or S1 and S2
International Advanced Level in Further Mathematics	FP1 and either FP2 or FP3	FP2, FP3, M1, M2, M3, S1, S2, S3, D1
International Advanced Level in Pure Mathematics	P1, P2, P3, P4, FP1	FP2 or FP3

The certification of each qualification requires **different** contributing units. For example, students who are awarded certificates in both International Advanced Level Mathematics and International Advanced Level Further Mathematics must use unit results from 12 **different** units, i.e. once a unit result has been used to cash in for a qualification, it cannot be re-used to cash in for another qualification.

Calculators

Calculators may be used in the examinations. Please see *Appendix 6: Use of calculators*.

Mathematics, Further Mathematics and Pure Mathematics content

Unit P1: Pure Mathematics 1

Compulsory unit for IAS Mathematics and Pure Mathematics
Compulsory unit for IAL Mathematics and Pure Mathematics

Externally assessed

P1.1 Unit description

Algebra and functions; coordinate geometry in the (x,y); trigonometry; differentiation; integration.

P1.2 Assessment information

1. Examination

- First assessment: January 2019.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

2. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet; *Mathematical Formulae and Statistical Tables*. This booklet will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Quadratic equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry

In the triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

area $= \frac{1}{2} ab \sin C$

arc length $= r\theta$

area of sector $= \frac{1}{2} r^2\theta$

2.	**Notation and formulae** *continued*	**Differentiation**	
		f(x)	**f′(x)**
		x^n	nx^{n-1}
		Integration	
		f(x)	$\int$ **f(x) dx**
		x^n	$\frac{1}{n+1}x^{n+1} + c, n \neq -1$

P1.3 Unit content

What students need to learn:		Guidance
1. Algebra and functions		
1.1	Laws of indices for all rational exponents.	$a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$ The equivalence of $a^{\frac{m}{n}}$ and $\sqrt[n]{a^m}$ should be known.
1.2	Use and manipulation of surds.	Students should be able to rationalise denominators.
1.3	Quadratic functions and their graphs.	
1.4	The discriminant of a quadratic function.	Need to know and to use $b^2 - 4ac > 0$, $b^2 - 4ac = 0$ and $b^2 - 4ac < 0$
1.5	Completing the square. Solution of quadratic equations.	Solution of quadratic equations by factorisation, use of the formula, use of a calculator and completing the square. $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$
1.6	Solve simultaneous equations; analytical solution by substitution.	
1.7	Interpret linear and quadratic inequalities graphically.	For example, $ax + b > cx + d$, $px^2 + qx + r \geqslant 0$, $px^2 + qx + r < ax + b$. Interpreting the third inequality as the range of x for which the curve $y = px^2 + qx + r$ is below the line with equation $y = ax + b$. Including inequalities with brackets and fractions. These would be reducible to linear or quadratic inequalities, e.g. $\frac{a}{x} < b$ becomes $ax < bx^2$, $x \neq 0$.
1.8	Represent linear and quadratic inequalities graphically.	Represent linear and quadratic inequalities such as $y > x + r$ and $y > ax^2 + bx + c$ graphically. Shading and use of dotted and solid line convention is required.
1.9	Solutions of linear and quadratic inequalities.	For example, solving $ax + b > cx + d$, $px^2 + qx + r \geqslant 0$, $px^2 + qx + r < ax + b$.
1.10	Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation.	Students should be able to use brackets. Factorisation of polynomials of degree n, $n \leqslant 3$, e.g. $x^3 + 4x^2 + 3x$. The notation $\mathrm{f}(x)$ may be used.

What students need to learn:		Guidance
1. Algebra and functions *continued*		
1.11	Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.	Functions to include simple cubic functions and the reciprocal functions $y = \frac{k}{x}$ and $y = \frac{k}{x^2}$ with $x \neq 0$. Knowledge of the term asymptote is expected. Also, trigonometric graphs.
1.12	Knowledge of the effect of simple transformations on the graph of $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$.	Students should be able to apply one of these transformations to any of the above functions (quadratics, cubics, reciprocals, sine, cosine, and tangent) and sketch the resulting graphs. Given the graph of any function $y = \mathrm{f}(x)$, students should be able to sketch the graph resulting from one of these transformations.
2. Coordinate geometry in the (x, y) plane		
2.1	Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$.	To include: (i) the equation of a line through two given points (ii) the equation of a line parallel (or perpendicular) to a given line through a given point. For example, the line perpendicular to the line $3x + 4y = 18$ through the point (2, 3) has equation $y - 3 = \frac{4}{3}(x - 2)$.
2.2	Conditions for two straight lines to be parallel or perpendicular to each other.	
3. Trigonometry		
3.1	The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab\sin C$.	Including the ambiguous case of the sine rule.
3.2	Radian measure, including use for arc length and area of sector.	Use of the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$.
3.3	Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.	Knowledge of graphs of curves with equations such as $y = 3\sin x$, $y = \sin\left(x + \frac{\pi}{6}\right)$, $y = \sin 2x$ is expected.

What students need to learn:		Guidance
4. Differentiation		
4.1	The derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.	For example, knowledge that $\frac{dy}{dx}$ is the rate of change of y with respect to x. Knowledge of the chain rule is not required. The notation $f'(x)$ and $f''(x)$ may be used.
4.2	Differentiation of x^n, and related sums, differences and constant multiples.	The ability to differentiate expressions such as $(2x + 5)(x - 1)$ and $\frac{x^2 + 5x - 3}{3\sqrt{x}}$ is expected.
4.3	Applications of differentiation to gradients, tangents and normals.	Use of differentiation to find equations of tangents and normals at specific points on a curve.
5. Integration		
5.1	Indefinite integration as the reverse of differentiation.	Students should know that a constant of integration is required.
5.2	Integration of x^n and related sums, differences and constant multiples.	(Excluding $n = -1$ and related sums, differences and multiples). For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$ and $\frac{(x+2)^2}{\sqrt{x}}$ is expected. Given $f'(x)$ and a point on the curve, students should be able to find an equation of the curve in the form $y = f(x)$.

Unit P2: Pure Mathematics 2

Compulsory unit for IAS Mathematics and Pure Mathematics
Compulsory unit for IAL Mathematics and Pure Mathematics

Externally assessed

P2.1 Unit description

Proof; algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; exponentials and logarithms; trigonometry; differentiation; integration.

P2.2 Assessment information

1. Prerequisites

A knowledge of the specification for P1 and its associated formulae is assumed and may be tested.

2. Examination

- First assessment: June 2019.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

2. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet; *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Laws of logarithms

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$

$k \log_a x \equiv \log_a (x^k)$

Trigonometry

$\sin^2 A + \cos^2 A \equiv 1$

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$

Area

area under a curve $= \int_a^b y \, \mathrm{d}x \;\; (y \geqslant 0)$

P2.3 Unit content

What students need to learn:		Guidance
1. Proof		
1.1	Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof stated below:	
1.2	Proof by exhaustion	Proof by exhaustion. This involves trying all the options. Suppose x and y are odd integers less than 7. Prove that their sum is divisible by 2.
1.3	Disproof by counter example.	Disproof by counter example – show that the statement "$n^2 - n + 1$ is a prime number for all values of n" is untrue.
2. Algebra and functions		
2.1	Simple algebraic division; use of the Factor Theorem and the Remainder Theorem.	Only division by $(ax + b)$ or $(ax – b)$ will be required, e.g. students should know that if $f(x) = 0$ when $x = \frac{b}{a}$, then $(ax – b)$ is a factor of $f(x)$. Students may be required to factorise cubic expressions such as $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$. Students should be familiar with the terms 'quotient' and 'remainder' and be able to determine the remainder when the polynomial $f(x)$ is divided by $(ax + b)$.
3. Coordinate geometry in the (x, y) plane		
3.1	Coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$ and including use of the following circle properties: (i) the angle in a semicircle is a right angle; (ii) the perpendicular from the centre to a chord bisects the chord; (iii) the perpendicularity of radius and tangent.	Students should be able to find the radius and the coordinates of the centre of the circle, given the equation of the circle and vice versa.

What students need to learn:		Guidance
4. Sequences and series		
4.1	Sequences, including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = \mathrm{f}(x_n)$.	
4.2	Understand and work with arithmetic sequences and series, including the formula for the nth term and the sum of a finite arithmetic series; the sum of the first n natural numbers.	The proof of the sum formula should be known. Understanding of $\sum$ notation will expected.
4.3	Increasing sequences, decreasing sequences and periodic sequences.	
4.4	Understand and work with geometric sequences and series, including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\lvert r\rvert < 1$.	For example, given the sum of a series students should be able to use logs to find the value of n. The proof of the sum formula for a finite series should be known. The sum to infinity may be expressed as S_∞.
4.5	Binomial expansion of $(a + bx)^n$ for positive integer n.	The notations $n!$, $\binom{n}{r}$ and ${}^nC_r$ may be used.
5. Exponentials and logarithms		
5.1	$y = a^x$ and its graph.	$a > 0$, $a \neq 1$
5.2	Laws of logarithms.	To include $\log_a (xy) \equiv \log_a x + \log_a y$, $\log_a \left(\frac{x}{y}\right) \equiv \log_a x - \log_a y$, $\log_a (x^k) \equiv k \log_a x$, $\log_a \left(\frac{1}{x}\right) \equiv -\log_a x$, $\log_a a = 1$ where $a, x, y > 0$, $a \neq 1$.
5.3	The solution of equations of the form $a^x = b$.	Students may use the change of base formula.

What students need to learn:		Guidance
6. Trigonometry		
6.1	Knowledge and use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, and $\sin^2\theta + \cos^2\theta = 1$.	
6.2	Solution of simple trigonometric equations in a given interval.	Students should be able to solve equations such as $\sin\left(x+\dfrac{\pi}{2}\right) = \frac{3}{4}$ for $0 < x < 2\pi$, $\cos(x + 30°) = \frac{1}{2}$ for $-180° < x < 180°$, $\tan 2x = 1$ for $90° < x < 270°$, $6\cos^2 x + \sin x - 5 = 0$ for $0 \leqslant x < 360°$, $\sin^2\left(x+\dfrac{\pi}{6}\right) = \frac{1}{2}$ for $-\pi \leqslant x < \pi$.
7. Differentiation		
7.1	Applications of differentiation to maxima and minima and stationary points, increasing and decreasing functions.	To include applications to curve sketching. Maxima and minima problems may be set in the context of a practical problem.
8. Integration		
8.1	Evaluation of definite integrals.	
8.2	Interpretation of the definite integral as the area under a curve.	Students will be expected to be able to evaluate the area of a region bounded by a curve and given straight lines. For example, find the finite area bounded by the curve $y = 6x - x^2$ and the line $y = 2x$. $\int x\,\mathrm{d}y$ will not be required. Students will be expected to be able to evaluate the area of a region bounded by two curves.
8.3	Approximation of area under a curve using the trapezium rule.	For example, use the trapezium rule to approximate $\int_0^1 \sqrt{(2x+1)}\,\mathrm{d}x$ using four strips. Use of increasing number of trapezia to improve accuracy and an estimate of the error may be required.

Unit P3: Pure Mathematics 3

Compulsory unit for IAL Mathematics and Pure Mathematics

Externally assessed

P3.1 Unit description

Algebra and functions; trigonometry; exponentials and logarithms; differentiation; integration; numerical methods.

P3.2 Assessment information

1. Prerequisites A knowledge of the specifications for P1 and P2, their prerequisites and associated formulae, is assumed and may be tested.

2. Examination

- First assessment: January 2020.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Trigonometry

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\mathrm{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2 \tan A}{1 - \tan^2 A}$

3. Notation and formulae *continued*

Differentiation

f(*x*)	**f′(*x*)**
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)\,g(x)$	$f'(x)\,g(x) + f(x)\,g'(x)$
$f(g(x))$	$f'(g(x))\,g'(x)$
a^x	$a^x \ln a$

Integration

f(*x*)	$\int f(x)\,dx$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\frac{1}{x}$	$\ln\lvert x \rvert + c,\ x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))\,g'(x)$	$f(g(x)) + c$
a^x	$\frac{a^x}{\ln a} + c$

P3.3 Unit content

What students need to learn:		Guidance
1. Algebra and functions		
1.1	Simplification of rational expressions including factorising and cancelling, and algebraic division.	Denominators of rational expressions will be linear or quadratic, e.g. $\dfrac{1}{ax+b}$, $\dfrac{ax+b}{px^2+qx+r}$, $\dfrac{x^3+1}{x^2-1}$.
1.2	Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.	The concept of a function as a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. The notation $\mathrm{f} : x \mapsto$ and $\mathrm{f}(x)$ will be used. Students should know that fg will mean 'do g first, then f'. Students should know that if f^{-1} exists, then $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x) = x$.
1.3	The modulus function.	Students should be able to sketch the graphs of $y = \lvert ax + b \rvert$ and the graphs of $y = \lvert \mathrm{f}(x) \rvert$ and $y = \mathrm{f}(\lvert x \rvert)$, given the graph of $y = \mathrm{f}(x)$. For example, sketch the graph with equation $y = \lvert 2x - 1 \rvert$ and use the graph to solve the equation $\lvert 2x - 1 \rvert = x + 5$ or the inequality $\lvert 2x - 1 \rvert > x + 5$.
1.4	Combinations of the transformations $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$.	Students should be able to sketch the graph of, for example, $y = 2\mathrm{f}(3x)$, $y = \mathrm{f}(-x) + 1$, given the graph of $y = \mathrm{f}(x)$ or the graph of, for example, $y = 3 + \sin 2x$, $y = -\cos\left(x + \dfrac{\pi}{4}\right)$. The graph of $y = \mathrm{f}(ax + b)$ will *not* be required.
2. Trigonometry		
2.1	Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.	Angles measured in both degrees and radians.
2.2	Knowledge and use of $\sec^2\theta = 1 + \tan^2\theta$ and $\mathrm{cosec}^2\theta = 1 + \cot^2\theta$.	

What students need to learn:		Guidance
2. Trigonometry *continued*		
2.3	Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm a)$ or $r\sin(\theta \pm a)$.	To include application to half angles. Knowledge of the t ($\tan\frac{1}{2}\theta$) formulae will *not* be required. Students should be able to solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval, and to prove identities such as $\cos x\cos 2x + \sin x\sin 2x \equiv \cos x$.
3. Exponential and logarithms		
3.1	The function e^x and its graph.	To include the graph of $y = e^{ax+b} + c$
3.2	The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x.	Solution of equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.
3.3	Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$.	Plot $\log y$ against $\log x$ and obtain a straight line where the intercept is $\log a$ and the gradient is n. Plot $\log y$ against x and obtain a straight line where the intercept is $\log k$ and the gradient is $\log b$.
4. Differentiation		
4.1	Differentiation of e^{kx}, $\ln kx$, $\sin kx$, $\cos kx$, $\tan kx$ and their sums and differences.	
4.2	Differentiation using the product rule, the quotient rule and the chain rule.	Differentiation of $\operatorname{cosec} x$, $\cot x$ and $\sec x$ are required. Skill will be expected in the differentiation of functions generated from standard functions using products, quotients and composition, such as $2x^4\sin x$, $\dfrac{e^{3x}}{x}$, $\cos x^2$ and $\tan^2 2x$.
4.3	The use of $\dfrac{dy}{dx} = \dfrac{1}{\left(\dfrac{dx}{dy}\right)}$	For example, finding $\dfrac{dy}{dx}$ for $x = \sin 3y$
4.4	Understand and use exponential growth and decay.	Students should be familiar with terms such as ‘initial’, ‘meaning when’ $t = 0$. Students may need to explore the behaviour for large values of t or to consider whether the range of values predicted is appropriate. Consideration of a second improved model may be required. Knowledge and use of the result $\dfrac{d}{dx}(a^x) = a^x\ln a$ is expected.

What students need to learn:		Guidance
5. Integration		
5.1	Integration of e^{kx}, $\frac{1}{x^n}$, $\sin kx$, $\cos kx$ and their sums and differences.	To include integration of standard functions such as $\sin 3x$, e^{5x}, $\frac{1}{2x}$
5.2	Integration by recognition of known derivatives to include integrals of the form $\int \frac{f'(x)}{f(x)}\,dx = \ln(f(x)) + c$ and $\int f'(x)[f(x)]^n\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$	For example, to include integration of $\tan x$, $\sec^2 2x$. Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$.
6. Numerical methods		
6.1	Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.	
6.2	Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$	Solution of equations by use of iterative procedures, for which leads will be given.

Unit P4: Pure Mathematics 4

Compulsory unit for IAL Mathematics and Pure Mathematics

Externally assessed

P4.1 Unit description

Proof; algebra and functions; coordinate geometry in the (x, y) plane; binomial expansion; differentiation; integration; vectors.

P4.2 Assessment information

1. Prerequisites

A knowledge of the specifications for P1, P2 and P3, their prerequisites and associated formulae, is assumed and may be tested.

2. Examination

- First assessment: June 2020.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This is a list of formulae that students are expected to remember and which will not be included in formulae booklets.

Vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$$

P4.3 Unit content

What students need to learn:		Guidance
1. Proof		
1.1	Proof by contradiction	Including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs.
2. Algebra and functions		
2.1	Decompose rational functions into partial fractions (denominators not more complicated than repeated linear terms).	Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$. The degree of the numerator may equal or exceed the degree of the denominator. Applications to integration, differentiation and series expansions. Quadratic factors in the denominator such as $(x^2 + a)$, $a > 0$, are *not* required.
3. Coordinate geometry in the (x, y) plane		
3.1	Parametric equations of curves and conversion between cartesian and parametric forms.	
4. Binomial expansion		
4.1	Binomial Series for any rational *n*.	For $\lvert x \rvert < \frac{b}{a}$, students should be able to obtain the expansion of $(ax + b)^n$, and the expansion of rational functions by decomposition into partial fractions.
5. Differentiation		
5.1	Differentiation of simple functions defined implicitly or parametrically.	The finding of equations of tangents and normals to curves given parametrically or implicitly is required.
5.2	Formation of simple differential equations.	Questions involving connected rates of change may be set.

What students need to learn:		Guidance
6. Integration		
6.1	Evaluation of volume of revolution.	$\pi \int y^2 \, dx$ is required, but *not* $\pi \int x^2 \, dy$. Students should be able to find a volume of revolution, given parametric equations.
6.2	Simple cases of integration by substitution and integration by parts. Understand these methods as the reverse processes of the chain and product rules respectively.	Students will be expected to use a substitution to find, e.g. $\int x\sqrt{x-2} \, dx$ The substitution will be given in more complicated integrals. The integral $\int \ln x \, dx$ is required. More than one application of integration by parts may be required, for example, $\int x^2 e^x \, dx$, $\int e^x \sin x \, dx$.
6.3	Simple cases of integration using partial fractions.	Integration of rational expressions such as those arising from partial fractions, e.g. $\frac{2}{3x+5}$, $\frac{3}{(x-1)^2}$. Note that the integration of other rational expressions, such as $\frac{x}{x^2+5}$ and $\frac{2}{(2x-1)^4}$ is also required (*see P3 section 5.2*).
6.4	Analytical solution of simple first order differential equations with separable variables.	General and particular solutions will be required.
6.5	Use integration to find the area under a curve given its parametric equations.	Students should be able to find the area under a curve given its parametric equations. Students will not be expected to sketch a curve from its parametric equations.
7. Vectors		
7.1	Vectors in two and three dimensions.	
7.2	Magnitude of a vector.	Students should be able to find a unit vector in the direction of **a**, and be familiar with $\lvert \mathbf{a} \rvert$.
7.3	Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.	
7.4	Position vectors.	$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

What students need to learn:		Guidance
7. Vectors *continued*		
7.5	The distance between two points.	The distance d between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$
7.6	Vector equations of lines.	To include the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t(\mathbf{d} - \mathbf{c})$ Conditions for two lines to be parallel, intersecting or skew.
7.7	The scalar product. Its use for calculating the angle between two lines.	Students should know that for $\overrightarrow{OA} = \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ and $\cos \angle AOB = \dfrac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ Students should know that if $\mathbf{a}.\mathbf{b} = 0$, and $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors, then $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

Unit FP1: Further Pure Mathematics 1

Compulsory unit for IAS Further Mathematics and Pure Mathematics

Compulsory unit for IAL Further Mathematics and Pure Mathematics

Externally assessed

FP1.1 Unit description

Complex numbers; roots of quadratic equations; numerical solution of equations; coordinate systems; matrix algebra; transformations using matrices; series; proof.

FP1.2 Assessment information

1. Prerequisites

A knowledge of the specification for P1 and P2, their prerequisites and associated formulae, is assumed and may be tested.

It is also necessary for students:

- to have a knowledge of location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval in which
- $f(x)$ is continuous
- to have a knowledge of rotating shapes through any angle about (0, 0)
- to be able to divide a cubic polynomial by a quadratic polynomial
- to be able to divide a quartic polynomial by a quadratic polynomial.

2. Examination

- First assessment: June 2019.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This is a list of formulae that students are expected to remember and which will not be included in formulae booklets.

Roots of quadratic equations

For $ax^2 + bx + c = 0$: $\alpha + \beta = -\dfrac{b}{a}$, $\alpha\beta = \dfrac{c}{a}$

Series

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$

FP1.3 Unit content

What students need to learn:		Guidance
1. Complex numbers		
1.1	Definition of complex numbers in the form $a + \mathrm{i}b$ and $r\cos\theta + \mathrm{i}\,r\sin\theta$.	The meaning of conjugate, modulus, argument, real part, imaginary part and equality of complex numbers should be known.
1.2	Sum, product and quotient of complex numbers.	$\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \lvert z_2 \rvert$ Knowledge of the result $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ is not required.
1.3	Geometrical representation of complex numbers in the Argand diagram. Geometrical representation of sums, products and quotients of complex numbers.	
1.4	Complex solutions of quadratic equations with real coefficients.	
1.5	Finding conjugate complex roots and a real root of a cubic equation with integer coefficients.	Knowledge that if z_1 is a root of $\mathrm{f}(z) = 0$ then z_1^* is also a root.
1.6	Finding conjugate complex roots and/or real roots of a quartic equation with real coefficients.	For example, (i) $\mathrm{f}(x) = x^4 - x^3 - 5x^2 + 7x + 10$ Given that $x = 2 + \mathrm{i}$ is a root of $\mathrm{f}(x) = 0$, use algebra to find the three other roots of $\mathrm{f}(x) = 0$ (ii) $\mathrm{g}(x) = x^4 - x^3 + 6x^2 + 14x - 20$ Given $\mathrm{g}(1) = 0$ and $\mathrm{g}(-2) = 0$, use algebra to solve $\mathrm{g}(x) = 0$ completely.

What students need to learn:		Guidance
2. Roots of quadratic equations		
2.1	Sum of roots and product of roots of a quadratic equation.	For the equation $ax^2 + bx + c = 0$, whose roots are α and β, then $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$.
2.2	Manipulation of expressions involving the sum of roots and product of roots.	Knowledge of the identity $\alpha^3 + \beta^3 \equiv (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$.
2.3	Forming quadratic equations with new roots.	For example, with roots α^3, β^3; $\frac{1}{\alpha}, \frac{1}{\beta}$; $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$; $\alpha + \frac{2}{\beta}$, $\beta + \frac{2}{\alpha}$; etc.
3. Numerical solution of equations		
3.1	Equations of the form $f(x) = 0$ solved numerically by: (i) interval bisection, (ii) linear interpolation, (iii) the Newton-Raphson process.	$f(x)$ will involve only functions used in P1 and P2. For the Newton-Raphson process, the only differentiation required will be as defined in unit P1 and P2.
4. Coordinate systems		
4.1	Cartesian equations for the parabola and rectangular hyperbola.	Students should be familiar with the equations: $y^2 = 4ax$ or $x = at^2$, $y = 2at$ and $xy = c^2$ or $x = ct$, $y = \frac{c}{t}$.
4.2	Idea of parametric equation for parabola and rectangular hyperbola.	The idea of $(at^2, 2at)$ as a general point on the parabola is all that is required.
4.3	The focus-directrix property of the parabola.	Concept of focus and directrix and parabola as locus of points equidistant from focus and directrix.
4.4	Tangents and normals to these curves.	Differentiation of $y = 2a^{\frac{1}{2}}x^{\frac{1}{2}}$, $y = \frac{c^2}{x}$. Parametric differentiation is not required.

What students need to learn:		Guidance
5. Matrix algebra integration		
5.1	Addition and subtraction of matrices.	
5.2	Multiplication of a matrix by a scalar.	
5.3	Products of matrices.	
5.4	Evaluation of 2 × 2 determinants.	Singular and non-singular matrices.
5.5	Inverse of 2 × 2 matrices.	Use of the relation $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.
6. Transformations using matrices		
6.1	Linear transformations of column vectors in two dimensions and their matrix representation.	The transformation represented by **AB** is the transformation represented by **B** followed by the transformation represented by **A**.
6.2	Applications of 2 × 2 matrices to represent geometrical transformations.	Identification and use of the matrix representation of single transformations from: reflection in coordinate axes and lines $y = \pm x$, rotation through any angle about (0, 0), stretches parallel to the x-axis and y-axis, and enlargement about centre (0, 0), with scale factor k, $(k \neq 0)$, where $k \in \mathbb{R}$.
6.3	Combinations of transformations.	Identification and use of the matrix representation of combined transformations.
6.4	The inverse (when it exists) of a given transformation or combination of transformations.	Idea of the determinant as an area scale factor in transformations.
7. Series		
7.1	Summation of simple finite series.	Students should be able to sum series such as $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$, $\sum_{r=1}^{n} r(r^2+2)$. The method of differences is not required.

What students need to learn:		Guidance
8. Proof		
8.1	Proof by mathematical induction.	To include induction proofs for (i) summation of series e.g. show $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ or $\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$ (ii) divisibility e.g. show $3^{2n} + 11$ is divisible by 4. (iii) finding general terms in a sequence e.g. if $u_{n+1} = 3u_n + 4$ with $u_1 = 1$, prove that $u_n = 3^n - 2$. (iv) matrix products e.g. show $\begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & -n \\ 9n & 3n+1 \end{pmatrix}$.

Unit FP2: Further Pure Mathematics 2

Optional unit for IAS Further Mathematics

Optional unit for IAL Further Mathematics and Pure Mathematics

Externally assessed

FP2.1 Unit description

	Inequalities; series; further complex numbers; first order differential equations; second order differential equations; Maclaurin and Taylor series; Polar coordinates.

FP2.2 Assessment information

1. Prerequisites	A knowledge of the specifications for P1, P2, P3, P4 and FP1, their prerequisites and associated formulae, is assumed and may be tested.
2. Examination	• First assessment: June 2020. • The assessment is 1 hour and 30 minutes. • The assessment is out of 75 marks. • Students must answer all questions. • Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*. • The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.
2. Notation	Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

FP2.3 Unit content

What students need to learn:		Guidance
1. Inequalities		
1.1	The manipulation and solution of algebraic inequalities and inequations, including those involving the modulus sign.	The solution of inequalities such as $\frac{1}{x-a} > \frac{x}{x-b}$, $\lvert x^2 - 1 \rvert > 2(x + 1)$.
2. Series		
2.1	Summation of simple finite series using the method of differences.	Students should be able to sum series such as $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ by using partial fractions such as $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$.
3. Further complex numbers		
3.1	Euler's relation $e^{i\theta} = \cos\theta + i\sin\theta$.	Students should be familiar with $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ and $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$.
3.2	De Moivre's theorem and its application to trigonometric identities and to roots of a complex number.	To include finding $\cos n\theta$ and $\sin m\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$ and also powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles. Students should be able to prove De Moivre's theorem for any integer n.
3.3	Loci and regions in the Argand diagram.	Loci such as $\lvert z - a \rvert = b$, $\lvert z - a \rvert = k\lvert z - b \rvert$, $\arg(z - a) = \beta$, $\arg\left(\frac{z-a}{z-b}\right) = \beta$ and regions such as $\lvert z - a \rvert \leqslant \lvert z - b \rvert$, $\lvert z - a \rvert \leqslant b$.
3.4	Elementary transformations from the z-plane to the w-plane.	Transformations such as $w = z^2$ and $w = \frac{az+b}{cz+d}$, where $a, b, c, d \in \mathbb{C}$, may be set.

What students need to learn:		Guidance
4. First order differential equations		
4.1	Further solution of first order differential equations with separable variables.	The formation of the differential equation may be required. Students will be expected to obtain particular solutions and also sketch members of the family of solution curves.
4.2	First order linear differential equations of the form $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x.	The integrating factor $e^{\int P\,dx}$ may be quoted without proof.
4.3	Differential equations reducible to the above types by means of a given substitution.	
5. Second order differential equations		
5.1	The linear second order differential equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ where a, b and c are real constants and the particular integral can be found by inspection or trial.	The auxiliary equation may have real distinct, equal or complex roots. $f(x)$ will have one of the forms ke^{px}, $A + Bx$, $p + qx + cx^2$ or $m\cos\omega x + n\sin\omega x$. Students should be familiar with the terms 'complementary function' and 'particular integral'. Students should be able to solve equations of the form $\frac{d^2y}{dx^2} + 4y = \sin 2x$.
5.2	Differential equations reducible to the above types by means of a given substitution.	
6. Maclaurin and Taylor series		
6.1	Third and higher order derivatives.	
6.2	Derivation and use of Maclaurin series.	The derivation of the series expansion of e^x, $\sin x$, $\cos x$, $\ln(1 + x)$ and other simple functions may be required.
6.3	Derivation and use of Taylor series.	The derivation, for example, of the expansion of $\sin x$ in ascending powers of $(x - \pi)$ up to and including the term in $(x - \pi)^3$.
6.4	Use of Taylor series method for series solutions of differential equations.	Students may, for example, be required to find the solution in powers of x as far as the term in x^4,of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, such that $y = 1$, $\frac{dy}{dx} = 0$ at $x = 0$.

What students need to learn:		Guidance
7. Polar coordinates		
7.1	Polar coordinates (r, θ), $r \geqslant 0$.	The sketching of curves such as $\theta = \alpha$, $r = p\sec(\alpha - \theta)$, $r = a$, $r = 2a\cos\theta$, $r = k\theta$, $r = a(1 \pm \cos\theta)$, $r = a(3 + 2\cos\theta)$, $r = a\cos 2\theta$ and $r^2 = a^2\cos 2\theta$ may be set.
7.1	Use of the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, \mathrm{d}\theta$ for area.	The ability to find tangents parallel to, or at right angles to, the initial line is expected.

Unit FP3: Further Pure Mathematics 3

Optional unit for IAS Further Mathematics

Optional unit for IAL Further Mathematics and Pure Mathematics

Externally assessed

FP3.1 Unit description

Hyperbolic functions; further coordinate systems; differentiation; integration; vectors; further matrix algebra.

FP3.2 Assessment information

1. Prerequisites	A knowledge of the specifications for P1, P2, P3, P4 and FP1, their prerequisites and associated formulae, is assumed and may be tested.
2. Examination	• First assessment: June 2020. • The assessment is 1 hour and 30 minutes. • The assessment is out of 75 marks. • Students must answer all questions. • Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*. • The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.
3. Notation	Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

FP3.3 Unit content

What students need to learn:		Guidance
1. Hyperbolic functions		
1.1	Definition of the six hyperbolic functions in terms of exponentials. Graphs and properties of the hyperbolic functions.	For example, $\cosh x = \frac{1}{2}(e^{x} + e^{-x})$, $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^{x} + e^{-x}}$. Students should be able to derive and use simple identities such as $\cosh^2 x - \sinh^2 x \equiv 1$ and $\cosh^2 x + \sinh^2 x \equiv \cosh 2x$ and to solve equations such as $a\cosh x + b\sinh x = c$.
1.2	Inverse hyperbolic functions, their graphs, properties and logarithmic equivalents.	For example, $\operatorname{arsinh} x = \ln\left[x + \sqrt{(1+x^2)}\right]$. Students may be required to prove this and similar results.
2. Further coordinate systems		
2.1	Cartesian and parametric equations for the ellipse and hyperbola.	Extension of work from FP1. Students should be familiar with the equations: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$; $x = a\cos t$, $y = b\sin t$. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$; $x = a\sec t$, $y = b\tan t$; $x = a\cosh t$, $y = b\sinh t$.
2.2	The focus-directrix properties of the ellipse and hyperbola, including the eccentricity.	For example, students should know that, for the ellipse, $b^2 = a^2(1 - e^2)$, the foci are $(ae, 0)$ and $(-ae, 0)$ and the equations of the directrices are $x = +\dfrac{a}{e}$ and $x = -\dfrac{a}{e}$.
2.3	Tangents and normals to these curves.	The condition for $y = mx + c$ to be a tangent to these curves is expected to be known.
2.4	Simple loci problems.	
3. Differentiation		
3.1	Differentiation of hyperbolic functions and expressions involving them.	For example, $\tanh 3x$, $x\sinh^2 x$, $\dfrac{\cosh 2x}{\sqrt{(x+1)}}$.
3.2	Differentiation of inverse functions, including trigonometric and hyperbolic functions.	For example, $\arcsin x + x\sqrt{(1 - x^2)}$, $\frac{1}{2}\operatorname{artanh} x^2$.

What students need to learn:		Guidance
4. Integration		
4.1	Integration of hyperbolic functions and expressions involving them.	
4.2	Integration of inverse trigonometric and hyperbolic functions.	For example, $\int \text{arsinh}\, x \, dx$, $\int \arctan x \, dx$.
4.3	Integration using hyperbolic and trigonometric substitutions.	To include the integrals of $\frac{1}{(a^2+x^2)}$, $\frac{1}{\sqrt{(a^2-x^2)}}$, $\frac{1}{\sqrt{(a^2+x^2)}}$, $\frac{1}{\sqrt{(x^2-a^2)}}$
4.4	Use of substitution for integrals involving quadratic surds.	In more complicated cases, substitutions will be given.
4.5	The derivation and use of simple reduction formulae.	Students should be able to derive formulae such as $nI_n = (n-1)I_{n-2},\ n \geqslant 2,$ for $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx$, $I_{n+2} = \frac{2\sin(n+1)x}{n+1} + I_n$ for $I_n = \int \frac{\sin nx}{\sin x} \, dx,\ n > 0.$
4.6	The calculation of arc length and the area of a surface of revolution.	The equation of the curve may be given in cartesian or parametric form. Equations in polar form will not be set.
5. Vectors		
5.1	The vector product $\mathbf{a} \times \mathbf{b}$ and the triple scalar product $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$.	The interpretation of $\lvert \mathbf{a} \times \mathbf{b} \rvert$ as an area and $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ as a volume.
5.2	Use of vectors in problems involving points, lines and planes. The equation of a line in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$.	Students may be required to use equivalent cartesian forms also. Applications to include (i) distance from a point to a plane, (ii) line of intersection of two planes, (iii) shortest distance between two skew lines.
5.3	The equation of a plane in the forms $\mathbf{r}.\mathbf{n} = p$, $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$.	Students may be required to use equivalent cartesian forms also.

What students need to learn:		Guidance
6. Further matrix algebra		
6.1	Linear transformations of column vectors in two and three dimensions and their matrix representation.	Extension of work from FP1 to 3 dimensions.
6.2	Combination of transformations. Products of matrices.	The transformation represented by $\mathbf{AB}$ is the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$.
6.3	Transpose of a matrix.	Use of the relation $(\mathbf{AB})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$.
6.4	Evaluation of 3×3 determinants.	Singular and non-singular matrices.
6.5	Inverse of 3×3 matrices.	Use of the relation $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.
6.6	The inverse (when it exists) of a given transformation or combination of transformations.	
6.7	Eigenvalues and eigenvectors of 2×2 and 3×3 matrices.	Normalised vectors may be required.
6.8	Reduction of symmetric matrices to diagonal form.	Students should be able to find an orthogonal matrix $\mathbf{P}$ such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP}$ is diagonal.

Unit M1: Mechanics 1

Optional unit for IAS Mathematics and Further Mathematics
Optional unit for IAL Mathematics and Further Mathematics

Externally assessed

M1.1 Unit description

Mathematical models in mechanics; vectors in mechanics; kinematics of a particle moving in a straight line; dynamics of a particle moving in a straight line or plane; statics of a particle; moments.

M1.2 Assessment information

1. Prerequisites A knowledge of P1 and P2 and associated formulae and of vectors in two dimensions.

2. Examination

- First assessment: June 2019.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Momentum = mv

Impulse = $mv - mu$

For constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

M1.3 Unit content

What students need to learn:		Guidance
1. Mathematical models in mechanics		
1.1	The basic ideas of mathematical modelling as applied in Mechanics.	Students should be familiar with the terms: particle, lamina, rigid body, rod (light, uniform, non-uniform), inextensible string, smooth and rough surface, light smooth pulley, bead, wire, peg. Students should be familiar with the assumptions made in using these models.
2. Vectors in mechanics		
2.1	Magnitude and direction of a vector. Resultant of vectors may also be required.	Students may be required to resolve a vector into two components or use a vector diagram. Questions may be set involving the unit vectors **i** and **j**.
2.2	Application of vectors to displacements, velocities, accelerations and forces in a plane.	Use of velocity = $\dfrac{\text{change of displacement}}{\text{time}}$ in the case of constant velocity, and of acceleration = $\dfrac{\text{change of velocity}}{\text{time}}$ in the case of constant acceleration, will be required.
3. Kinematics of a particle moving in a straight line		
3.1	Motion in a straight line with constant acceleration.	Graphical solutions may be required, including displacement-time, velocity-time, speed-time and acceleration-time graphs. Knowledge and use of formulae for constant acceleration will be required.
4. Dynamics of a particle moving in a straight line or plane		
4.1	The concept of a force. Newton's laws of motion.	Simple problems involving constant acceleration in scalar form or as a vector of the form $a\mathbf{i} + b\mathbf{j}$.
4.2	Simple applications including the motion of two connected particles.	Problems may include (i) the motion of two connected particles moving in a straight line or under gravity when the forces on each particle are constant; problems involving smooth fixed pulleys and/or pegs may be set (ii) motion under a force which changes from one fixed value to another, e.g. a particle hitting the ground (iii) motion directly up or down a smooth or rough inclined plane.
4.3	Momentum and impulse. The impulse-momentum principle. The principle of conservation of momentum applied to two particles colliding directly.	Knowledge of Newton's law of restitution is not required. Problems will be confined to those of a one-dimensional nature.
4.4	Coefficient of friction.	An understanding of $F = \mu R$ when a particle is moving.

What students need to learn:		Guidance
5. Statics of a particle		
5.1	Forces treated as vectors. Resolution of forces.	
5.2	Equilibrium of a particle under coplanar forces. Weight, normal reaction, tension and thrust, friction.	Only simple cases of the application of the conditions for equilibrium to uncomplicated systems will be required.
5.3	Coefficient of friction.	An understanding of $F \leqslant \mu R$ in a situation of equilibrium.
6. Moments		
6.1	Moment of a force.	Simple problems involving coplanar parallel forces acting on a body and conditions for equilibrium in such situations.

Unit M2: Mechanics 2

Optional unit for IAS Further Mathematics
Optional unit for IAL Mathematics and Further Mathematics

Externally assessed

M2.1 Unit description

Kinematics of a particle moving in a straight line or plane; centres of mass; work and energy; collisions; statics of rigid bodies.

M2.1 Assessment information

1. Prerequisites

A knowledge of the specifications for P1, P2, P3, P4 and M1, and their prerequisites and associated formulae, is assumed and may be tested.

2. Examination

- First assessment: June 2020.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Kinetic energy = $\frac{1}{2}mv^2$

Potential energy = mgh

M2.3 Unit content

What students need to learn:		Guidance
1. Kinematics of a particle moving in a straight line or plane		
1.1	Motion in a vertical plane with constant acceleration, e.g. under gravity.	
1.2	Simple cases of motion of a projectile.	
1.3	Velocity and acceleration when the displacement is a function of time.	The setting up and solution of equations of the form $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{f}(t)$ or $\frac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{g}(t)$ will be consistent with the level of calculus in P1, P2 P3 and P4.
1.4	Differentiation and integration of a vector with respect to time.	For example, given that $\mathbf{r} = t^2\mathbf{i} + t^{\frac{3}{2}}\mathbf{j}$, find $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ at a given time.
2. Centres of mass		
2.1	Centre of mass of a discrete mass distribution in one and two dimensions.	
2.2	Centre of mass of uniform plane figures, and simple cases of composite plane figures.	The use of an axis of symmetry will be acceptable where appropriate. Use of integration is not required. Figures may include the shapes referred to in the formulae book. Results given in the formulae book may be quoted without proof.
2.3	Simple cases of equilibrium of a plane lamina.	The lamina may (i) be suspended from a fixed point; (ii) be free to rotate about a fixed horizontal axis; (iii) be put on an inclined plane.
3. Work and energy		
3.1	Kinetic and potential energy, work and power. The work-energy principle. The principle of conservation of mechanical energy.	Problems involving motion under a constant resistance and/or up and down an inclined plane may be set.

What students need to learn:		Guidance
4. Collisions		
4.1	Momentum as a vector. The impulse-momentum principle in vector form. Conservation of linear momentum.	
4.2	Direct impact of elastic particles. Newton's law of restitution. Loss of mechanical energy due to impact.	Students will be expected to know and use the inequalities $0 \leqslant e \leqslant 1$ (where e is the coefficient of restitution).
4.3	Successive impacts of up to three particles or two particles and a smooth plane surface.	Collision with a plane surface will not involve oblique impact.
5. Statics of rigid bodies		
5.1	Moment of a force.	
5.2	Equilibrium of rigid bodies.	Problems involving parallel and non-parallel coplanar forces. Problems may include rods or ladders resting against smooth or rough vertical walls and on smooth or rough ground.

Unit M3: Mechanics 3

Optional unit for IAS Further Mathematics
Optional unit for IAL Further Mathematics

Externally assessed

M3.1 Unit description

Further kinematics; elastic strings and springs; further dynamics; motion in a circle; statics of rigid bodies.

M3.2 Assessment information

1. Prerequisites

A knowledge of the specifications for P1, P2, P3, P4 and M1 and M2, and their prerequisites and associated formulae, is assumed and may be tested.

2. Examination

- First assessment: June 2020.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

The tension in an elastic string = $\dfrac{\lambda x}{l}$

The energy stored in an elastic string = $\dfrac{\lambda x^2}{2l}$

For SHM:

$\ddot{x} = -\omega^2 x$,

$x = a\cos\omega t$ or $x = a\sin\omega t$,

$v^2 = \omega^2(a^2 - x^2)$,

$T = \dfrac{2\pi}{\omega}$

M3.3 Unit content

What students need to learn:		Guidance
1. Further kinematics		
1.1	Kinematics of a particle moving in a straight line when the acceleration is a function of the displacement (x), or time (t).	The setting up and solution of equations where $\frac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{f}(t)$, $v\frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{f}(x)$, $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{f}(x)$ or $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{f}(t)$ will be consistent with the level of calculus required in units P1, P2, P3 and P4.
2. Elastic strings and springs		
2.1	Elastic strings and springs. Hooke's law.	
2.2	Energy stored in an elastic string or spring.	Simple problems using the work-energy principle involving kinetic energy, potential energy and elastic energy.
3. Further dynamics		
3.1	Newton's laws of motion, for a particle moving in one dimension, when the applied force is variable.	The solution of the resulting equations will be consistent with the level of calculus in units P1, P2, P3 and P4. Problems may involve the law of gravitation, i.e. the inverse square law.
3.2	Simple harmonic motion.	Proof that a particle moves with simple harmonic motion in a given situation may be required (i.e. showing that $\ddot{x} = -\omega^2 x$). Geometric or calculus methods of solution will be acceptable. Students will be expected to be familiar with standard formulae, which may be quoted without proof.
3.3	Oscillations of a particle attached to the end of an elastic string or spring.	Oscillations will be in the direction of the string or spring only.
4. Motion in a circle		
4.1	Angular speed.	
4.2	Radial acceleration in circular motion. The forms $r\omega^2$ and $\frac{v^2}{r}$ are required.	
4.3	Uniform motion of a particle moving in a horizontal circle.	Problems involving the 'conical pendulum', an elastic string, motion on a banked surface, as well as other contexts, may be set.
4.4	Motion of a particle in a vertical circle.	

What students need to learn:		Guidance
5. Statics of rigid bodies		
5.1	Centre of mass of uniform rigid bodies and simple composite bodies.	The use of integration and /or symmetry to determine the centre of mass of a uniform body will be required.
5.2	Simple cases of equilibrium of rigid bodies.	To include (i) suspension of a body from a fixed point, (ii) a rigid body placed on a horizontal or inclined plane.

Unit S1: Statistics 1

Optional unit for IAS Mathematics and Further Mathematics
Optional unit for IAL Mathematics and Further Mathematics

Externally assessed

S1.1 Unit description

Mathematical models in probability and statistics; representation and summary of data; probability; correlation and regression; discrete random variables; discrete distributions; the Normal distribution.

S1.2 Assessment information

1. Examination

- First assessment: June 2019.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

2. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Mean = $\bar{x} = \frac{\sum x}{n}$ or $\frac{\sum fx}{\sum f}$

Standard deviation = $\sqrt{(\text{variance})}$

Interquartile range = IQR = $Q_3 - Q_1$

$P(A') = 1 - P(A)$

For independent events A and B,

$P(B \mid A) = P(B)$, $P(A \mid B) = P(A)$,

$P(A \cap B) = P(A)\,P(B)$

$E(aX + b) = aE(X) + b$

$\text{Var}(aX + b) = a^2\,\text{Var}(X)$

2. Notation and formulae *continued*

Cumulative distribution function for a discrete random variable:

$$F(x_0) = P(X \leqslant x_0) = \sum_{x \leqslant x_0} p(x)$$

Standardised Normal Random Variable $Z = \dfrac{X - \mu}{\sigma}$

where $X \sim N(\mu, \sigma^2)$

S1.3 Unit content

What students need to learn:		Guidance
1.	**Mathematical models in probability and statistics**	
1.1	The basic ideas of mathematical modelling as applied in probability and statistics.	
2.	**Representation and summary of data**	
2.1	Histograms, stem and leaf diagrams, box plots.	Using histograms, stem and leaf diagrams and box plots to compare distributions. Back-to-back stem and leaf diagrams may be required. Drawing of histograms, stem and leaf diagrams or box plots will not be the direct focus of examination questions.
2.2	Measures of location – mean, median, mode.	Calculation of mean, mode and median, range and interquartile range will not be the direct focus of examination questions. Students will be expected to draw simple inferences and give interpretations to measures of location and dispersion. Significance tests will not be expected. Data may be discrete, continuous, grouped or ungrouped. Understanding and use of coding.
2.3	Measures of dispersion – variance, standard deviation, range and interpercentile ranges.	Simple interpolation may be required. Interpretation of measures of location and dispersion.
2.4	Skewness. Concepts of outliers.	Students may be asked to illustrate the location of outliers on a box plot. Any rule to identify outliers will be specified in the question.
3.	**Probability**	
3.1	Elementary probability.	
3.2	Sample space. Exclusive and complementary events. Conditional probability.	Understanding and use of $P(A') = 1 - P(A)$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A \cap B) = P(A)\,P(B \mid A)$.
3.3	Independence of two events.	$P(B \mid A) = P(B)$, $P(A \mid B) = P(A)$, $P(A \cap B) = P(A)\,P(B)$.
3.4	Sum and product laws.	Use of tree diagrams and Venn diagrams. Sampling with and without replacement.

What students need to learn:		Guidance
4. Correlation and regression		
4.1	Scatter diagrams. Linear regression.	Calculation of the equation of a linear regression line using the method of least squares. Students may be required to draw this regression line on a scatter diagram.
4.2	Explanatory (independent) and response (dependent) variables. Applications and interpretations.	Use to make predictions within the range of values of the explanatory variable and the dangers of extrapolation. Derivations will not be required. Variables other than x and y may be used. Linear change of variable may be required.
4.3	The product moment correlation coefficient, its use, interpretation and limitations.	Derivations and tests of significance will not be required.
5. Discrete random variables		
5.1	The concept of a discrete random variable.	
5.2	The probability function and the cumulative distribution function for a discrete random variable.	Simple uses of the probability function $\mathrm{p}(x)$ where $\mathrm{p}(x) = \mathrm{P}(X = x)$. Use of the cumulative distribution function: $\mathrm{F}(x_0) = \mathrm{P}(X \leqslant x_0) = \sum_{x \leqslant x_0} \mathrm{p}(x)$.
5.3	Mean and variance of a discrete random variable.	Use of $\mathrm{E}(X)$, $\mathrm{E}(X^2)$ for calculating the variance of X. Knowledge and use of $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$, $\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X)$.
5.4	The discrete uniform distribution.	The mean and variance of this distribution.
6. The Normal distribution		
5.1	The Normal distribution including the mean, variance and use of tables of the cumulative distribution function.	Knowledge of the shape and the symmetry of the distribution is required. Knowledge of the probability density function is not required. Derivation of the mean, variance and cumulative distribution function is not required. Interpolation is not necessary. Questions may involve the solution of simultaneous equations.

Unit S2: Statistics 2

Optional unit for IAS Further Mathematics
Optional unit for IAL Mathematics and Further Mathematics

Externally assessed

S2.1 Unit description

The Binomial and Poisson distributions; continuous random variables; continuous distributions; samples; hypothesis tests.

S2.2 Assessment information

1. Prerequisites

A knowledge of the specification for S1 and its prerequisites and associated formulae, together with a knowledge of differentiation and integration of polynomials, binomial coefficients in connection with the binomial distribution and the evaluation of the exponential function, is assumed and may be tested.

2. Examination

- First assessment: June 2020.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This is a list of formulae that students are expected to remember and which will not be included in formulae booklets.

For the continuous random variable X having probability density function $\mathrm{f}(x)$,

$$\mathrm{P}(a < X \leqslant b) = \int_a^b \mathrm{f}(x)\,\mathrm{d}x$$

$$\mathrm{f}(x) = \frac{\mathrm{dF}(x)}{\mathrm{d}x}$$

S2.3 Unit content

What students need to learn:		Guidance
1. The Binomial and Poisson distributions		
1.1	The binomial and Poisson distributions.	Students will be expected to use these distributions to model a real-world situation and to comment critically on their appropriateness. Cumulative probabilities by calculation or by reference to tables. Students will be expected to use the additive property of the Poisson distribution – e.g. if the number of events per minute ~ Po(λ) then the number of events per 5 minutes ~ Po(5λ).
1.2	The mean and variance of the binomial and Poisson distributions.	No derivations will be required.
1.3	The use of the Poisson distribution as an approximation to the binomial distribution.	
2. Continuous random variables		
2.1	The concept of a continuous random variable.	
2.2	The probability density function and the cumulative distribution function for a continuous random variable.	Use of the probability density function f(x), where $\mathrm{P}(a < X \leqslant b) = \int_a^b \mathrm{f}(x)\,\mathrm{d}x$. Use of the cumulative distribution function $\mathrm{F}(x_0) = \mathrm{P}(X \leqslant x_0) = \int_{-\infty}^{x_0} \mathrm{f}(x)\,\mathrm{d}x$. The formulae used in defining f(x) will be restricted to simple polynomials which may be expressed piecewise.
2.3	Relationship between density and distribution functions.	$\mathrm{f}(x) = \dfrac{\mathrm{dF}(x)}{\mathrm{d}x}$.
2.4	Mean and variance of continuous random variables.	
2.5	Mode, median and quartiles of continuous random variables.	
3. Continuous distributions		
3.1	The continuous uniform (rectangular) distribution.	Including the derivation of the mean, variance and cumulative distribution function.
3.2	Use of the Normal distribution as an approximation to the binomial distribution and the Poisson distribution, with the application of the continuity correction.	

What students need to learn:		Guidance
4. Hypothesis tests		
4.1	Population, census and sample. Sampling unit, sampling frame.	Students will be expected to know the advantages and disadvantages associated with a census and a sample survey.
4.2	Concepts of a statistic and its sampling distribution.	
4.3	Concept and interpretation of a hypothesis test. Null and alternative hypotheses.	Use of hypothesis tests for refinement of mathematical models.
4.4	Critical region.	Use of a statistic as a test statistic.
4.5	One-tailed and two-tailed tests.	
4.6	Hypothesis tests for the parameter p of a binomial distribution and for the mean of a Poisson distribution.	Students are expected to know how to use tables to carry out these tests. Questions may also be set not involving tabular values. Tests for the parameter p of a binomial distribution may involve the use of a normal approximation to calculate probabilities.

Unit S3: Statistics 3

Optional unit for IAS Further Mathematics
Optional unit for IAL Further Mathematics

Externally assessed

S3.1 Unit description

Combinations of random variables; sampling; estimation, confidence intervals and tests; goodness of fit and contingency tables; regression and correlation.

S3.2 Assessment information

1. Prerequisites

A knowledge of the specifications for S1 and S2, and their prerequisites and associated formulae, is assumed and may be tested.

2. Examination

- First assessment: June 2020.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

$aX \pm bY \sim \mathrm{N}(a\mu_x \pm b\mu_y,\ a^2\sigma_x^2 + b^2\sigma_y^2)$ where X and Y are independent and $X \sim \mathrm{N}(\mu_x, \sigma_x^2)$ and $Y \sim \mathrm{N}(\mu_y, \sigma_y^2)$.

S3.3 Unit content

What students need to learn:		Guidance
1. Combinations of random variables		
1.1	Distribution of linear combinations of independent Normal random variables.	If $X \sim \mathrm{N}(\mu_x, \sigma_x^2)$ and $Y \sim \mathrm{N}(\mu_y, \sigma_y^2)$ independently, then $aX \pm bY \sim \mathrm{N}(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2)$. No proofs required.
2. Sampling		
2.1	Methods for collecting data. Simple random sampling. Use of random numbers for sampling.	
2.2	Other methods of sampling: stratified, systematic, quota.	The circumstances in which they might be used. Their advantages and disadvantages.
3. Estimation, confidence intervals and tests		
3.1	Concepts of standard error, estimator, bias.	The sample mean, $\bar{x}$, and the sample variance, $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, as unbiased estimates of the corresponding population parameters.
3.2	The distribution of the sample mean $\bar{X}$.	$\bar{X}$ has mean μ and variance $\frac{\sigma^2}{n}$. If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$. No proofs required.
3.3	Concept of a confidence interval and its interpretation.	Link with hypothesis tests.
3.4	Confidence limits for a Normal mean, with variance known.	Students will be expected to know how to apply the Normal distribution and use the standard error and obtain confidence intervals for the mean, rather than be concerned with any theoretical derivations.
3.5	Hypothesis tests for the mean of a Normal distribution with variance known.	Use of $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$.
3.6	Use of Central Limit theorem to extend hypothesis tests and confidence intervals to samples from non-Normal distributions. Use of large sample results to extend to the case in which the variance is unknown.	$\frac{\bar{X} - \mu}{S / \sqrt{n}}$ can be treated as $\mathrm{N}(0, 1)$ when n is large. A knowledge of the t-distribution is not required.

What students need to learn:		Guidance
3. Estimation, confidence intervals and tests *continued*		
3.7	Hypothesis test for the difference between the means of two Normal distributions with variances known.	Use of $\dfrac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x}+\dfrac{\sigma_y^2}{n_y}}} \sim \mathrm{N}(0, 1)$.
3.8	Use of large sample results to extend to the case in which the population variances are unknown.	Use of $\dfrac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\dfrac{S_x^2}{n_x}+\dfrac{S_y^2}{n_y}}} \sim \mathrm{N}(0, 1)$. A knowledge of the *t*-distribution is not required.
4. Goodness of fit and contingency tables		
4.1	The null and alternative hypotheses. The use of $\sum_{i=1}^{n}\dfrac{(O_i-E_i)^2}{E_i}$ as an approximate χ^2 statistic.	Applications to include the discrete uniform, binomial, Normal, Poisson and continuous uniform (rectangular) distributions. Lengthy calculations will not be required.
4.2	Degrees of freedom.	Students will be expected to determine the degrees of freedom when one or more parameters are estimated from the data. Cells should be combined when $E_i < 5$. Yates' correction is not required.
5. Regression and correlation		
5.1	Spearman's rank correlation coefficient, its use, interpretation and limitations.	Numerical questions involving ties will not be set. Some understanding of how to deal with ties will be expected.
5.2	Testing the hypothesis that a correlation is zero.	Use of tables for Spearman's and product moment correlation coefficients.

Unit D1: Decision Mathematics 1

Optional unit for IAS Mathematics and Further Mathematics
Optional unit for IAL Mathematics and Further Mathematics

Externally assessed

D1.1 Unit description

Algorithms; algorithms on graphs; algorithms on graphs II; critical path analysis; linear programming.

D1.2 Assessment information

1. Preamble	Students should be familiar with the terms defined in the *Glossary for D1*. Students should show clearly how an algorithm has been applied. Matrix representation will be required but matrix manipulation is not required. Students will be required to model and interpret situations, including cross-checking between models and reality.
2. Examination	• First assessment: June 2019. • The assessment is 1 hour and 30 minutes. • The assessment is out of 75 marks. • Students must answer all questions. • Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*. • The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.
3. Notation and formulae	Students will be expected to understand the symbols outlined in *Appendix 7: Notation*. Students are expected to know any other formulae that might be required and which are **not** included in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

D1.3 Unit content

What students need to learn:		Guidance
1. Algorithms		
1.1	The general ideas of algorithms and the implementation of an algorithm given by a flow chart or text.	The order of an algorithm is not expected. Whenever finding the middle item of any list, the method defined in the glossary must be used.
1.2	Students should be familiar with bin packing, bubble sort, quick sort, binary search.	When using the quick sort algorithm, the pivot should be chosen as the middle item of the list.
2. Algorithms on graphs		
2.1	The minimum spanning tree (minimum connector) problem. Prim's and Kruskal's algorithm.	Matrix representation for Prim's algorithm is expected. Drawing a network from a given matrix and writing down the matrix associated with a network will be involved.
2.2	Dijkstra's algorithm for finding the shortest path.	
3. Algorithms on graphs II		
3.1	Algorithm for finding the shortest route around a network, travelling along every edge at least once and ending at the start vertex. The network will have up to four odd nodes.	Also known as the 'Chinese postman' problem. Students will be expected to use inspection to consider all possible pairings of odd nodes. (The application of Floyd's algorithm to the odd nodes is not required.)
3.2	The practical and classical Travelling Salesman problems. The classical problem for complete graphs satisfying the triangle inequality.	The use of short cuts to improve upper bound is included.
3.3	Determination of upper and lower bounds using minimum spanning tree methods.	The conversion of a network into a complete network of shortest 'distances' is included.
3.4	The nearest neighbour algorithm.	

What students need to learn:		Guidance
4. Critical path analysis		
4.1	Modelling of a project by an activity network, from a precedence table.	Activity on arc will be used. The use of dummies is included. In a precedence network, precedence tables will only show immediate predecessors.
4.2	Completion of the precedence table for a given activity network.	
4.3	Algorithm for finding the critical path. Earliest and latest event times. Earliest and latest start and finish times for activities.	
4.4	Total float. Gantt (cascade) charts. Scheduling.	
5. Linear programming		
5.1	Formulation of problems as linear programs.	
5.2	Graphical solution of two variable problems using ruler and vertex methods.	
5.3	Consideration of problems where solutions must have integer values.	

Glossary for D1

1. Algorithms

In a list containing N items the 'middle' item has position $[\frac{1}{2}(N+1)]$ if N is odd, $[\frac{1}{2}(N+2)]$ if N is even, so that if $N = 9$, the middle item is the 5th and if $N = 6$ it is the 4th.

2. Algorithms on graphs

A **graph** G consists of points (**vertices** or **nodes**) which are connected by lines (**edges** or **arcs**).

A **subgraph** of G is a graph, each of whose vertices belongs to G and each of whose edges belongs to G.

If a graph has a number associated with each edge (usually called its **weight**) then the graph is called a **weighted graph** or **network**.

The **degree** or **valency** of a vertex is the number of edges incident to it. A vertex is **odd** (**even**) if it has **odd** (**even**) degree.

A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, and in which no vertex appears more than once.

A **cycle** (**circuit**) is a closed path, i.e. the end vertex of the last edge is the start vertex of the first edge.

Two vertices are **connected** if there is a path between them. A graph is **connected** if all its vertices are connected.

If the edges of a graph have a direction associated with them they are known as **directed edges** and the graph is known as a **digraph**.

A **tree** is a connected graph with no cycles.

A **spanning tree** of a graph G is a subgraph which includes all the vertices of G and is also a tree.

A **minimum spanning tree** (MST) is a spanning tree such that the total length of its arcs is as small as possible. (MST is sometimes called a **minimum connector**.)

A graph in which each of the n vertices is connected to every other vertex is called a **complete graph**.

The **travelling salesman problem** is 'find a route of minimum length which visits every vertex in an undirected network'. In the '**classical**' problem, each vertex is visited once only. In the '**practical**' problem, a vertex may be revisited.

For three vertices A, B and C, the **triangular inequality** is 'length $AB \leqslant$ length AC + length CB', where AB is a longest length.

A **walk** in a network is a finite sequence of edges such that the end vertex of one edge is the start vertex of the next.

A walk which visits every vertex, returning to its starting vertex, is called a **tour**.

3. Critical path analysis

The **total float** $F(i, j)$ of activity (i, j) is defined to be $F(i, j) = l_j - e_i$ – duration (i, j), where e_i is the earliest time for event i and l_j is the latest time for event j.

Assessment information

Assessment requirements

The Pearson Edexcel International Advanced Subsidiary in Mathematics, Further Mathematics and Pure Mathematics consist of three externally-examined units.

The Pearson Edexcel International Advanced Level Mathematics, Further Mathematics and Pure Mathematics consist of six externally-examined units.

Please see the *Assessment availability and first award* section for information on when the assessment for each unit will be available from.

Unit	IAS or IA2	Assessment information	Number of raw marks allocated in the unit
P1: Pure Mathematics 1	IAS	Written examination	75 marks
P2: Pure Mathematics 2	IAS		
P3: Pure Mathematics 3	IA2		
P4: Pure Mathematics 4	IA2		
FP1: Further Mathematics 1	IAS		
FP2: Further Mathematics 2	IA2		
FP3: Further Mathematics 3	IA2		
M1: Mechanics 1	IAS		
M2: Mechanics 2	IA2		
M3: Mechanics 3	IA2		
S1: Statistics 1	IAS		
S2: Statistics 2	IA2		
S3: Statistics 3	IA2		
D1: Decision Mathematics 1	IAS		

Sample assessment materials

Sample papers and mark schemes can be found in the *Pearson Edexcel International Advanced Subsidiary/Advanced Level in Mathematics, Further Mathematics and Pure Mathematics Sample Assessment Materials (SAMs)* document.

Assessment objectives and weightings

		Minimum weighting in IAS	Minimum weighting in IA2	Minimum weighting in IAL
AO1	Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.	30%	30%	30%
AO2	Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.	30%	30%	30%
AO3	Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.	10%	10%	10%
AO4	Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.	5%	5%	5%
AO5	Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%	5%	5%

Relationship of assessment objectives to units

All figures in the following table are expressed as marks out of 75.

Unit number	Assessment objective				
	AO1	**AO2**	**AO3**	**AO4**	**AO5**
P1	30–35	25–30	5–15	5–10	1–5
P2	25–30	25–30	5–10	5–10	5–10
P3	25–30	25–30	5–10	5–10	5–10
P4	25–30	25–30	5–10	5–10	5–10
FP1	25–30	25–30	0–5	5–10	5–10
FP2	25–30	25–30	0–5	7–12	5–10
FP3	25–30	25–30	0–5	7–12	5–10
M1	20–25	20–25	15–20	6–11	4–9
M2	20–25	20–25	10–15	7–12	5–10
M3	20–25	25–30	10–15	5–10	5–10
S1	20–25	20–25	15–20	5–10	5–10
S2	25–30	20–25	10–15	5–10	5–10
S3	25–30	20–25	10–15	5–10	5–10
D1	20–25	20–25	15–20	5–10	5–10

Assessment availability and first award

Unit number	January 2019	June 2019	October 2019	January 2020	June 2020	October 2020	January 2021
P1	✓	✓	✓	✓	✓	✓	✓
P2	×	✓	✓	✓	✓	✓	✓
P3	×	×	×	✓	✓	✓	✓
P4	×	×	×	×	✓	✓	✓
FP1	×	✓	×	✓	✓	×	✓
FP2	×	×	×	×	✓	×	✓
FP3	×	×	×	×	✓	×	✓
M1	×	✓	✓	✓	✓	✓	✓
M2	×	×	×	×	✓	✓	✓
M3	×	×	×	×	✓	×	✓
S1	×	✓	✓	✓	✓	✓	✓
S2	×	×	×	×	✓	✓	✓
S3	×	×	×	×	✓	×	✓
D1	×	✓	×	✓	✓	×	✓
IAS Mathematics award	×	✓	✓	✓	✓	✓	✓
IAS Further Mathematics award	×	✓	×	✓	✓	✓	✓
IAS Pure Mathematics award	×	✓	×	✓	✓	✓	✓
IAL Mathematics award	×	×	×	×	✓	✓	✓
IAL Further Mathematics award	×	×	×	×	✓	✓	✓
IAL Pure Mathematics award	×	×	×	×	✓	✓	✓

From June 2020, **all** units will be assessed in January and June and **just** units P1, P2, P3, P4, M1, M2, S1 and S2 in October, for the lifetime of the qualifications.

From June 2020, **all IAL and IAS qualifications will be awarded** in January, June and October for the lifetime of the qualifications.

Administration and general information

Entries and resitting of units

Entries

Details of how to enter students for the examinations for these qualifications can be found in our *International Information Manual*. A copy is made available to all examinations officers and is available on our website, qualifications.pearson.com.

Resitting of units

Students can resit any unit irrespective of whether the qualification is to be cashed in. If a student resits a unit more than once, the best available unit result will count towards the final grade.

Access arrangements, reasonable adjustments, special consideration and malpractice

Equality and fairness are central to our work. Our equality policy requires all students to have equal opportunity to access our qualifications and assessments, and our qualifications to be awarded in a way that is fair to every student.

We are committed to making sure that:

- students with a protected characteristic (as defined by the UK Equality Act 2010) are not, when they are undertaking one of our qualifications, disadvantaged in comparison to students who do not share that characteristic
- all students achieve the recognition they deserve for undertaking a qualification and that this achievement can be compared fairly to the achievement of their peers.

Language of assessment

Assessment of these qualifications will be available in English only. All student work must be in English.

Access arrangements

Access arrangements are agreed before an assessment. They allow students with special educational needs, disabilities or temporary injuries to:

- access the assessment
- show what they know and can do without changing the demands of the assessment.

The intention behind an access arrangement is to meet the particular needs of an individual student with a disability without affecting the integrity of the assessment. Access arrangements are the principal way in which awarding bodies comply with the duty under the Equality Act 2010 to make 'reasonable adjustments'.

Access arrangements should always be processed at the start of the course. Students will then know what is available and have the access arrangement(s) in place for assessment.

Reasonable adjustments

The Equality Act 2010 requires an awarding organisation to make reasonable adjustments where a student with a disability would be at a substantial disadvantage in undertaking an assessment. The awarding organisation is required to take reasonable steps to overcome that disadvantage.

A reasonable adjustment for a particular student may be unique to that individual and therefore might not be in the list of available access arrangements.

Whether an adjustment will be considered reasonable will depend on a number of factors, including:

- the needs of the student with the disability
- the effectiveness of the adjustment
- the cost of the adjustment; and
- the likely impact of the adjustment on the student with the disability and other students.

An adjustment will not be approved if it involves unreasonable costs to the awarding organisation, timeframes or affects the security or integrity of the assessment. This is because the adjustment is not 'reasonable'.

Special consideration

Special consideration is a post-examination adjustment to a student's mark or grade to reflect temporary injury, illness or other indisposition at the time of the examination/assessment, which has had, or is reasonably likely to have had, a material effect on a candidate's ability to take an assessment or demonstrate their level of attainment in an assessment.

Further information

Please see our website for further information about how to apply for access arrangements and special consideration.

For further information about access arrangements, reasonable adjustments and special consideration please refer to the JCQ website: www.jcq.org.uk.

Candidate malpractice

Candidate malpractice refers to any act by a candidate that compromises or seeks to compromise the process of assessment or which undermines the integrity of the qualifications or the validity of results/certificates.

Candidate malpractice in examinations **must** be reported to Pearson using a *JCQ Form M1* (available at www.jcq.org.uk/exams-office/malpractice). The form should be emailed to candidatemalpractice@pearson.com. Please provide as much information and supporting documentation as possible. Note that the final decision regarding appropriate sanctions lies with Pearson.

Failure to report malpractice constitutes staff or centre malpractice.

Staff/centre malpractice

Staff and centre malpractice includes both deliberate malpractice and maladministration of our qualifications. As with candidate malpractice, staff and centre malpractice is any act that compromises or seeks to compromise the process of assessment or which undermines the integrity of the qualifications or the validity of results/certificates.

All cases of suspected staff malpractice and maladministration **must** be reported immediately, before any investigation is undertaken by the centre, to Pearson on a *JCQ Form M2(a)* (available at www.jcq.org.uk/exams-office/malpractice).

The form, supporting documentation and as much information as possible should be emailed to pqsmalpractice@pearson.com. Note that the final decision regarding appropriate sanctions lies with Pearson.

Failure to report malpractice itself constitutes malpractice.

More-detailed guidance on malpractice can be found in the latest version of the document *JCQ General and Vocational Qualifications Suspected Malpractice in Examinations and Assessments Policies and Procedures,* available at www.jcq.org.uk/exams-office/malpractice.

Awarding and reporting

The Pearson Edexcel International Advanced Subsidiary in Mathematics, Further Mathematics and Pure Mathematics will be graded on a five-grade scale from A to E. The Pearson Edexcel International Advanced Level in Mathematics, Further Mathematics and Pure Mathematics will be graded on a six-point scale A* to E. Individual unit results will be reported.

A pass in an International Advanced Subsidiary subject is indicated by one of the five grades A, B, C, D, E, of which grade A is the highest and grade E the lowest. A pass in an International Advanced Level subject is indicated by one of the six grades A*, A, B, C, D, E, of which grade A* is the highest and grade E the lowest. Students whose level of achievement is below the minimum judged by Pearson to be of sufficient standard to be recorded on a certificate will receive an unclassified U result.

Information on certification opportunities for the Pearson Edexcel International Advanced Subsidiary/Advanced in Mathematics, Further Mathematics and Pure Mathematics is given in the section *Assessment availability and first award*.

Unit results

Students will receive a uniform mark between 0 and the maximum uniform mark for each unit.

The uniform marks at each grade threshold for each unit are:

Unit grade	**Maximum uniform mark**	**A**	**B**	**C**	**D**	**E**
	100	**80**	**70**	**60**	**50**	**40**

Qualification results

The minimum uniform marks required for each grade:

International Advanced Subsidiary (cash-in code: XMA01, XFM01, XPM01)

Qualification grade	**Maximum uniform mark**	**A**	**B**	**C**	**D**	**E**
	300	**240**	**210**	**180**	**150**	**120**

Students with a uniform mark in the range 0–119 will be Unclassified (U).

International Advanced Level (cash-in code: YMA01, YFM01, YPM01)

Qualification grade	**Maximum uniform mark**	**A**	**B**	**C**	**D**	**E**
	600	**480**	**420**	**360**	**300**	**240**

Students with a uniform mark in the range 0–239 will be Unclassified (U).

For International Advanced Level in Mathematics, A* will be awarded to students who have achieved grade A overall (at least 480 of the 600 maximum uniform mark) and at least 180 of the 200 combined maximum uniform mark for the P3 and P4 units.

For International Advanced Level in Further Mathematics, A* will be awarded to students who have achieved a grade A overall (at least 480 of the 600 maximum uniform mark) and at least 270 of the 300 combined maximum uniform mark for their **best** three IA2 units (whether pure or application units).

For International Advanced Level in Pure Mathematics, A* will be awarded to students who have achieved a grade A overall (at least 480 of the 600 maximum uniform mark) and at least 270 of the 300 combined maximum uniform mark for their IA2 units.

Student recruitment and progression

Pearson follows the JCQ policy concerning recruitment to our qualifications in that:

- they must be available to anyone who is capable of reaching the required standard
- they must be free from barriers that restrict access and progression
- equal opportunities exist for all students.

Prior learning and other requirements

There are no prior learning or other requirements for these qualifications.

Students who would benefit most from studying these qualifications are likely to have a Level 2 qualification such as an International GCSE in Mathematics.

Progression

Students can progress from these qualifications to:

- a range of different, relevant academics or vocational higher education qualifications
- employment in a relevant sector
- further training.

Appendices

Appendix 1: Codes

Type of code	Use of code	Code
Unit codes	Each unit is assigned a unit code. This unit code is used as an entry code to indicate that a student wishes to take the assessment for that unit. Centres will need to use the entry codes only when entering students for their examination.	Unit P1: WMA11/01 Unit P2: WMA12/01 Unit P3: WMA13/01 Unit P4: WMA14/01 Unit FP1: WFM01/01 Unit FP2: WFM02/01 Unit FP3: WFM03/01 Unit M1: WME01/01 Unit M2: WME02/01 Unit M3: WME03/01 Unit S1: WST01/01 Unit S2: WST02/01 Unit S3: WST03/01 Unit D1: WDM11/01
Cash in codes	The cash-in code is used as an entry code to aggregate the student's unit scores to obtain the overall grade for the qualification. Centres will need to use the entry codes only when entering students for their qualification.	**International Advanced Subsidiary:** Mathematics – XMA01 Further Mathematics – XFM01 Pure Mathematics – XPM01 **International Advanced Level:** Mathematics – YMA01 Further Mathematics – YFM01 Pure Mathematics – YPM01
Entry codes	The entry codes are used to: • enter a student for the assessment of a unit • aggregate the student's unit scores to obtain the overall grade for the qualification.	Please refer to the *Pearson Information Manual*, available on our website.

Appendix 2: Pearson World Class Qualification design principles

Pearson's World Class Qualification design principles mean that all Edexcel qualifications are developed to be **rigorous, demanding, inclusive and empowering**.

We work collaboratively to gain approval from an external panel of educational thought-leaders and assessment experts from across the globe. This is to ensure that Edexcel qualifications are globally relevant, represent world-class best practice in qualification and assessment design, maintain a consistent standard and support learner progression in today's fast-changing world.

Pearson's Expert Panel for World-Class Qualifications is chaired by Sir Michael Barber, a leading authority on education systems and reform. He is joined by a wide range of key influencers with expertise in education and employability.

"I'm excited to be in a position to work with the global leaders in curriculum and assessment to take a fresh look at what young people need to know and be able to do in the 21st century, and to consider how we can give them the opportunity to access that sort of education." Sir Michael Barber.

Endorsement from Pearson's Expert Panel for World Class Qualifications for the International Advanced Subsidiary (IAS)/International Advanced Level (IAL) development process

May 2014

"We were chosen, either because of our expertise in the UK education system, or because of our experience in reforming qualifications in other systems around the world as diverse as Singapore, Hong Kong, Australia and a number of countries across Europe.

We have guided Pearson through what we judge to be a rigorous world class qualification development process that has included, where appropriate:

- extensive international comparability of subject content against the highest-performing jurisdictions in the world
- benchmarking assessments against UK and overseas providers to ensure that they are at the right level of demand
- establishing External Subject Advisory Groups, drawing on independent subject-specific expertise to challenge and validate our qualifications.

Importantly, we have worked to ensure that the content and learning is future oriented, and that the design has been guided by Pearson's Efficacy Framework. This is a structured, evidenced process which means that learner outcomes have been at the heart of this development throughout.

We understand that ultimately it is excellent teaching that is the key factor to a learner's success in education but as a result of our work as a panel we are confident that we have supported the development of Edexcel IAS and IAL qualifications that are outstanding for their coherence, thoroughness and attention to detail and can be regarded as representing world-class best practice."

Sir Michael Barber (Chair)
Chief Education Advisor, Pearson plc

Professor Lee Sing Kong
Dean and Managing Director, National Institute of Education International, Singapore

Dr Peter Hill
Former Chief Executive ACARA

Bahram Bekhradnia
President, Higher Education Policy Institute

Professor Jonathan Osborne
Stanford University

Dame Sally Coates
Director of Academies (South), United Learning Trust

Professor Dr Ursula Renold
Federal Institute of Technology, Switzerland

Professor Bob Schwartz
Harvard Graduate School of Education

Professor Janice Kay
Provost, University of Exeter

Jane Beine
Head of Partner Development, John Lewis Partnership

Jason Holt
CEO, Holts Group

All titles correct as at May 2014

Appendix 3: Transferable skills

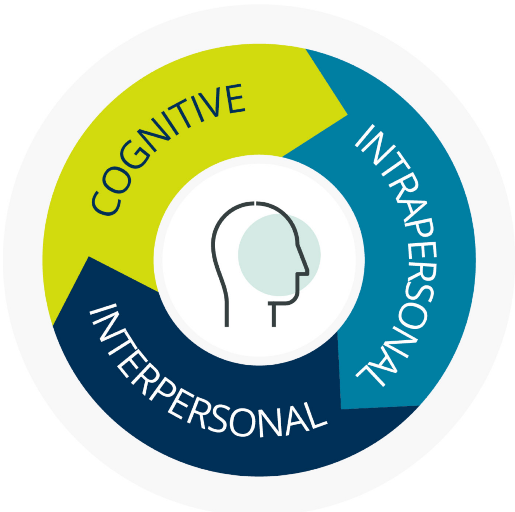
The need for transferable skills

In recent years, higher-education institutions and employers have consistently flagged the need for students to develop a range of transferable skills to enable them to respond with confidence to the demands of undergraduate study and the world of work.

The Organisation for Economic Co-operation and Development (OECD) defines skills, or competencies, as 'the bundle of knowledge, attributes and capacities that can be learned and that enable individuals to successfully and consistently perform an activity or task and can be built upon and extended through learning.'[1]

To support the design of our qualifications, the Pearson Research Team selected and evaluated seven global 21st-century skills frameworks. Following on from this process, we identified the National Research Council's (NRC) framework [2] as the most evidence-based and robust skills framework, and have used this as a basis for our adapted skills framework.

The framework includes cognitive, intrapersonal skills and interpersonal skills.



The skills have been interpreted for this specification to ensure they are appropriate for the subject. All of the skills listed are evident or accessible in the teaching, learning and/or assessment of the qualifications. Some skills are directly assessed. Pearson materials will support you in identifying these skills and developing these skills in students.

The table below sets out the framework and gives an indication of the skills that can be found in mathematics and indicates the interpretation of the skill in this area. A full subject interpretation of each skill, with mapping to show opportunities for student development is given on the subject pages of our website: qualifications.pearson.com

---

[1] OECD – *Better Skills, Better Jobs, Better Lives* (OECD Publishing, 2012)
[2] Koenig J A, National Research Council – *Assessing 21st Century Skills: Summary of a Workshop* (National Academies Press, 2011)

Cognitive skills	Cognitive processes and strategies	• Critical thinking • Problem solving • Analysis • Reasoning/argumentation • Interpretation • Decision making • Adaptive learning • Executive function
	Creativity	• Creativity • Innovation
Intrapersonal skills	Intellectual openness	• Adaptability • Personal and social responsibility • Intellectual interest and curiosity • Continuous learning
	Work ethic/ conscientiousness	• Initiative • Self-direction • Responsibility • Perseverance • Productivity • Self-regulation (metacognition, forethought, reflection) • Ethics • Integrity
	Positive core self-evaluation	• Self-monitoring/self-evaluation/self-reinforcement
Interpersonal skills	Teamwork and collaboration	• Communication • Collaboration • Teamwork • Cooperation • Interpersonal skills • Empathy/perspective taking • Negotiation
	Leadership	• Leadership • Responsibility • Assertive communication • Self-presentation

Assimilating given information, or data found, determining the goal and establishing at least one route from one to the other. Combining information in what is often a multi-step process. Drawing diagrams and introducing helpful notation. Selecting the most efficient route to the solution from other possible routes. Checking that the answer is reasonable. Selecting and realising an elegant solution.

Reflecting on ideas learnt, problems met, whether solved or not, and using these to improve on one's own performance in the future. Taking an active role in planning one's own learning. Setting goals and meeting them in a continually developing fashion. Understanding how ideas, such as rigorous logical thought, can be applied to future decision-making situations, used to assess the credibility of arguments and validate claims.

Use mathematics as an effective medium of communication of ideas, concepts and solutions to problems. Use questioning skills appropriately to elicit further information or information needed.

Appendix 4: Level 3 Extended Project qualification

What is the Extended Project?

The Extended Project is a standalone qualification that can be taken alongside International Advanced Level (IAL) qualifications. It supports the development of independent learning skills and helps to prepare students for their next step – whether that be higher education or employment. The qualification:

- is recognised by higher education for the skills it develops
- is worth half of an International Advanced Level (IAL) qualification at grades A*–E
- carries UCAS points for university entry.

The Extended Project encourages students to develop skills in the following areas: research, critical thinking, extended writing and project management. Students identify and agree a topic area of their choice for in-depth study (which may or may not be related to an IAL subject they are already studying), guided by their teacher.

Students can choose from one of four approaches to produce:

- a dissertation (for example an investigation based on predominately secondary research)
- an investigation/field study (for example a practical experiment)
- a performance (for example in music, drama or sport)
- an artefact (for example creating a sculpture in response to a client brief or solving an engineering problem).

The qualification is coursework/non-examined assessment and students are assessed on the skills of managing, planning and evaluating their project. Students will research their topic, develop skills to review and evaluate the information, and then present the final outcome of their project.

The Extended Project has 120 guided learning hours (GLH) consisting of a 40-GLH taught element that includes teaching the technical skills (for example research skills) and an 80-GLH guided element that includes mentoring students through the project work. The qualification is 100% internally assessed and externally moderated.

How to link the Extended Project with mathematics

The Extended Project creates the opportunity to develop transferable skills for progression to higher education and to the workplace through the exploration of either an area of personal interest or a topic of interest from within the mathematics qualification content.

Through the Extended Project, students will develop skills that support their study of mathematics, including:

- conducting, organising and using research
- independent reading in the subject area
- planning, project management and time management
- defining a hypothesis to be tested in investigations or developing a design brief
- collecting, handling and interpreting data and evidence
- evaluating arguments and processes, including arguments in favour of alternative interpretations of data and evaluation of experimental methodology
- critical thinking.

In the context of the Extended Project, critical thinking refers to the ability to identify and develop arguments for a point of view or hypothesis and to consider and respond to alternative arguments.

Types of Extended Project related to mathematics

Students may produce a dissertation on any topic that can be researched and argued. In mathematics this might involve working on a substantial statistical project or a project that requires the use of mathematical modelling.

Projects can give students the opportunity to develop mathematical skills that cannot be adequately assessed in examination questions.

- ***Statistics*** – students can have the opportunity to plan a statistical enquiry project, use different methods of sampling and data collection, use statistical software packages to process and investigate large quantities of data and review results to decide if more data is needed.
- ***Mathematical modelling*** – students can have the opportunity to choose modelling assumptions, compare with experimental data to assess the appropriateness of their assumptions and refine their modelling assumptions until they get the required accuracy of results.

Using the Extended Project to support breadth and depth

In the Extended Project, students are assessed on the quality of the work they produce and the skills they develop and demonstrate through completing this work. Students should demonstrate that they have extended themselves in some significant way beyond what they have been studying in mathematics. Students can demonstrate extension in one or more dimensions:

- ***deepening understanding*** – where a student explores a topic in greater depth than in the specification content. This could be an in-depth exploration of one of the topics in the specification
- ***broadening skills*** – where a student learns a new skill. This might involve learning the skills in statistics or mathematical modelling mentioned above or learning a new mathematical process and its practical uses
- ***widening perspectives*** – where the student's project spans different subjects. Projects in a variety of subjects need to be supported by data and statistical analysis. Students studying mathematics with design and technology can carry out design projects involving the need to model a situation mathematically in planning their design.

A wide range of information to support the delivery and assessment of the Extended Project, including the specification, teacher guidance for all aspects, an editable scheme of work and exemplars for all four approaches, can be found on our website.

Appendix 5: Glossary

Term	Definition
Assessment objectives	The requirements that students need to meet to succeed in the qualification. Each assessment objective has a unique focus, which is then targeted in examinations or non-examined assessment. Assessment objectives may be assessed individually or in combination.
External assessment	An examination that is held at the same time and place in a global region.
International Advanced Subsidiary	Abbreviated to IAS.
International Advanced Level	Abbreviated to IAL.
Linear	Linear qualifications have all assessments at the end of a course of study. It is not possible to take one assessment earlier in the course of study.
Modular	Modular qualifications contain units of assessment. These units can be taken during the course of study. The final qualification grade is worked out from the combined unit results.
Raw marks	Raw marks are the actual marks that students achieve when taking an assessment. When calculating an overall grade, raw marks often need to be converted so that it is possible to see the proportionate achievement of a student across all units of study.
Uniform Mark Scale (UMS)	Student actual marks (or raw marks) will be converted into a UMS mark so that it is possible to see the proportionate result of a student. Two units may each be worth 25% of a total qualification. The raw marks for each unit may differ, but the uniform mark will be the same.
Unit	A modular qualification will be divided into a number of units. Each unit will have its own assessment.

Appendix 6: Use of calculators

Students may use a calculator in assessments for these qualifications. Centres are responsible for making sure that calculators used by their students meet the requirements given in the table below.

Students are expected to have available a calculator with at least the following keys: $+$, $-$, $\times$, $\div$, π, x^2, $\sqrt{x}$, $\frac{1}{x}$, x^y, $\ln x$, e^x, $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory.

Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Students must be familiar with the requirements before their assessments for these qualifications.

Calculators must be:	**Calculators must not:**
• of a size suitable for use on a desk • either battery or solar powered • free of lids, cases and covers that contain printed instructions or formulae.	• be designed or adapted to offer any of these facilities: ○ language translators ○ symbolic algebraic manipulation ○ symbolic differentiation or integration ○ communication with other machines or the internet • be borrowed from another candidate during an examination for any reason* • have retrievable information stored in them. This includes: ○ databanks ○ dictionaries ○ mathematical formulae ○ text.
The candidate is responsible for the following: • the calculator's power supply • the calculator's working condition • clearing anything stored in the calculator.	

*An invigilator may give a student a calculator.

Further information can be found in the JCQ documents *Instructions for conducting examinations* and *Information for candidates for written examinations*, available at www.jcq.org.uk/exams-office.

Appendix 7: Notation

The following notation will be used in the examinations.

1. Set notation		
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.4	$\{x : \ldots\}$	the set of all x such that $\ldots$
1.5	$\mathrm{n}(A)$	the number of elements in set A
1.6	$\varnothing$	the empty set
1.7	ε	the universal set
1.8	A'	the complement of the set A
1.9	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.10	$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
1.11	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, \ldots, n-1\}$
1.13	$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in Z,\ q \in Z^+\right\}$
1.14	$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
1.15	$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geqslant 0\}$
1.16	$\mathbb{R}$	the set of real numbers
1.17	$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
1.18	$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geqslant 0\}$
1.19	$\mathbb{C}$	the set of complex numbers
1.20	(x, y)	the ordered pair x, y
1.21	$A \times B$	the cartesian product of sets A and B, i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
1.22	$\subseteq$	is a subset of
1.22	$\subset$	is a proper subset of
1.23	$\cup$	union
1.24	$\cap$	intersection

1. Set notation *continued*		
1.25	$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leqslant x \leqslant b\}$
1.26	$[a, b), [a, b[$	the interval $\{x \in \mathbb{R} : a \leqslant x < b\}$
1.27	$(a, b],]a, b]$	the interval $\{x \in \mathbb{R} : a < x \leqslant b\}$
1.28	$(a, b),]a, b[$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
1.29	$y\,R\,x$	y is related to x by the relation R
1.30	$y \sim x$	y is equivalent to x, in the context of some equivalence relation
2. Miscellaneous symbols		
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	$\cong$	is isomorphic to
2.6	$\propto$	is proportional to
2.7	$<$	is less than
2.8	$\leqslant, \ngtr$	is less than or equal to, is not greater than
2.9	$>$	is greater than
2.10	$\geqslant, \nless$	is greater than or equal to, is not less than
2.11	∞	infinity
2.12	$p \wedge q$	p and q
2.13	$p \vee q$	p or q (or both)
2.14	$\sim p$	not p
2.15	$p \Rightarrow q$	p implies q (if p then q)
2.16	$p \Leftarrow q$	p is implied by q (if q then p)
2.17	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.18	$\exists$	there exists
2.19	$\forall$	for all

3. Operations		
3.1	$a + b$	a plus b
3.2	$a - b$	a minus b
3.3	$a \times b$, ab, $a.b$	a multiplied by b
3.4	$a \div b$, $\frac{a}{b}$, a/b	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7	$\sqrt{a}$	the positive square root of a
3.8	$\lvert a \rvert$	the modulus of a
3.9	$n!$	n factorial
3.10	$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n \in \mathbb{Z}^+$, $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$
4. Functions		
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
4.3	$\mathrm{f}: x \to y$	the function f maps the element x to the element y
4.4	f^{-1}	the inverse function of the function f
4.5	$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
4.6	$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.7	Δx, δx	an increment of x
4.8	$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.9	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
4.10	$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{n}(x)$	the first, second, ... , nth derivatives of $\mathrm{f}(x)$ with respect to x
4.11	$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x

4. **Functions** *continued*		
4.12	$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
4.13	$\dfrac{\partial V}{\partial x}$	the partial derivative of V with respect to x
4.14	$\dot{x},\ \ddot{x},\ \ldots$	the first, second, ... derivatives of x with respect to t
5. Exponential and logarithmic functions		
5.1	e	base of natural logarithms
5.2	e^x, $\exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x$, $\log_\mathrm{e} x$	natural logarithm of x
5.5	$\lg x$, $\log_{10} x$	logarithm of x to base 10
6. Circular and hyperbolic functions		
6.1	$\left.\begin{array}{l}\sin, \cos, \tan, \\ \operatorname{cosec}, \sec, \cot\end{array}\right\}$	the circular functions
6.2	$\left.\begin{array}{l}\arcsin, \arccos, \arctan, \\ \operatorname{arccosec}, \operatorname{arcsec}, \operatorname{arccot}\end{array}\right\}$	the inverse circular functions
6.3	$\left.\begin{array}{l}\sinh, \cosh, \tanh, \\ \operatorname{cosech}, \operatorname{sech}, \coth\end{array}\right\}$	the hyperbolic functions
6.4	$\left.\begin{array}{l}\operatorname{arsinh}, \operatorname{arcosh}, \operatorname{artanh}, \\ \operatorname{arcosech}, \operatorname{arsech}, \operatorname{arcoth}\end{array}\right\}$	the inverse hyperbolic functions
7. Complex numbers		
7.1	i, j	square root of -1
7.2	z	a complex number, $z = x + \mathrm{i}y$
7.3	$\operatorname{Re} z$	the real part of z, $\operatorname{Re} z = x$
7.4	$\operatorname{Im} z$	the imaginary part of z, $\operatorname{Im} z = y$
7.5	$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{\left(x^2 + y^2\right)}$
7.6	$\arg z$	the argument of z, $\arg z = \theta, -\pi < x \leqslant \pi$
7.7	z^*	the complex conjugate of z, $x - \mathrm{i}y$

8. Matrices		
8.1	$\mathbf{M}$	a matrix $\mathbf{M}$
8.2	$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
8.3	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
8.4	det $\mathbf{M}$ or $\lvert\mathbf{M}\rvert$	the determinant of the square matrix $\mathbf{M}$
9. Vectors		
9.1	$\mathbf{a}$	the vector $\mathbf{a}$
9.2	$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
9.3	$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
9.4	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
9.5	$\lvert\mathbf{a}\rvert$, a	the magnitude of $\mathbf{a}$
9.6	$\lvert\overrightarrow{AB}\rvert$, AB	the magnitude of $\overrightarrow{AB}$
9.7	$\mathbf{a}.\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
9.8	$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$
10. Probability and statistics		
10.1	$A, B, C,$ etc.	events
10.2	$A \cup B$	union of the events A and B
10.3	$A \cap B$	intersection of the events A and B
10.4	$\mathrm{P}(A)$	probability of the event A
10.5	A'	complement of the event A
10.6	$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
10.7	X, Y, R, etc.	random variables
10.8	x, y, r, etc	values of the random variables $X, Y, R,$ etc
10.9	$x_1, x_2, \ldots$	observations
10.11	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
10.12	$\mathrm{p}(x)$	probability function $\mathrm{P}(X = x)$ of the discrete random variable X
10.13	$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
10.14	$\mathrm{f}(x), \mathrm{g}(x), \ldots$	the value of the probability density function of a continuous random variable X

10.	**Probability and statistics** *continued*	
10.15	$\mathrm{F}(x)$, $\mathrm{G}(x)$, …	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of a continuous random variable X
10.16	$\mathrm{E}(X)$	expectation of the random variable X
10.17	$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
10.18	$\mathrm{Var}(X)$	variance of the random variable X
10.19	$\mathrm{G}(t)$	probability generating function for a random variable which takes the values 0, 1, 2, …
10.20	$\mathrm{B}(n, p)$	binomial distribution with parameters n and p
10.21	$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
10.22	μ	population mean
10.23	σ^2	population variance
10.24	σ	population standard deviation
10.25	$\bar{x}$, m	sample mean
10.26	s^2, $\hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$
10.27	ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
10.28	Φ	corresponding cumulative distribution function
10.29	ρ	product moment correlation coefficient for a population
10.30	r	product moment correlation coefficient for a sample
10.31	$\mathrm{Cov}\,(X, Y)$	covariance of X and Y

FOR INFORMATION ABOUT EDEXCEL, BTEC OR LCCI QUALIFICATIONS
VISIT QUALIFICATIONS.PEARSON.COM

PEARSON EDUCATION LIMITED. REGISTERED IN ENGLAND AND WALES NO. 872828
REGISTERED OFFICE: 80 STRAND, LONDON WC2R 0RL
VAT REG NO GB 278 537121

GETTY IMAGES: ALEX BELMONLINSKY

Pearson
Edexcel

ISBN 978-1-4469-4981-8

9 781446 949818